A Mathematical model to predict the US Airlines operation costs and airports charges per route per passenger

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Abstract

A mathematical model to estimate the average airlines operational costs and airports charges per route is important for airlines companies trying to open new routes and for data generation for other purpose such as transport modeling, simulation modeling, investment analyses for airlines and airports, etc. In this paper, a mathematical model is proposed for calculating airline operations costs and airports charges on a certain route by a specific airline. This model was set up by analyzing the domestic US air transport market. A multi-regression analysis determined the relation between distance, airline operations costs and airports charges between origin and destination and market fares. The model calculates the average airlines operations costs and airports charges per route by using these costs as parameters to estimates airlines routes fares. Thus, Airlines and airports cost factors are estimated by least square method between real fares data per route and the proposed model estimations. The results have been validated comparing with real airlines operations costs data showing a correlation over 99%, meaning that the average airlines operational costs and airports charges per route can be estimated by the model using real fares data and also that route fares are mainly determined by airline operations costs and airport charges.

Keywords
Airfare pricing, Airfare pricing determinants, Airline business, Airport business, Airline Airport relationship
1 Introduction

The global air transportation system has shown an exponential growth since the beginning of the 19th century (Radnoti, 2001). This growth was mainly affected by new airline business models, wars (Iraq and Afghanistan), and illnesses (Severe Acute Respiratory Syndrome, SARS), which increase airline and airport operational costs (Neufville and Odoni, 2003).

Since the deregulation and liberalization (privatization) of the air transportation industry in the 1970’s (Aldamari and Fagan, 2005), the competition between airlines has increased and domestic yields have decreased (Guillen and Ashish, 2004). This made airline operational costs and airport charges important factors to dominate routes, increase market share and passenger volume. The differences between airlines operational costs, competition and willingness of consumers to change to another carrier are main factors that cause different route fares (Borestein and Rose, 1994). Thus, to be able to compete and widen their air traffic market, airlines have improved their business models applying different business strategies, such as full-service carriers (FSC’s) or low-cost carriers (LCC’s), to reduce airline and airport operational costs, drop down fares and maximize profits.

In a non-competing airline scenario the interest for FSC’s to minimize operations cost did not exist. The increments of operations cost were paid by passengers. Airlines accepted each other tickets, service were concentrated in few hub airports and few slots were available at the hub airports. FSC’s enjoyed the government regulation that control airport regulations (Barret, 2004). Nowadays, FSC’s have been forced to put attention in the minimization of airline operation costs and reduce fares to be more competitive in a new scenario, mainly affected in short-haul operations by the LCC’s.

FSC’s are applying LCC strategies to low their costs, be more profitable and lower fares as an advantage. But also they are using different strategies (Hunter, 2006) that are opposite to the LCC models such as the expansion of the share of capacity allocation to high fare and business class by increasing seating quality.

Airlines operate from different airports according to their strategies. This has a direct effect on airline fares. Airports charge airline fees for operating routes as an origin or destination. Thus, airlines have formed strategies to fly from different types of airport taking advantages of lower airport costs (Forsyth, 2007) thus affecting airline fares.

In a very competitive industry, the estimation of the average airlines operational costs and airport charges between two different cities is very helpful for airlines to make a decision whether or not to open new routes. Knowledge of the average airline operational costs and airport charges allows them to be more competitive and prevent to enter in those routes, where according with their operational costs and the competition costs, where they have advantages or disadvantages to be successful.

The literature shows different mathematical models using different parameters mainly developed to estimate airlines fares. Pels and Rietveld (2004) developed models to estimate fares for different airlines. They found that FSC’s do not follow the fare movements of LCC’s. Some carriers appear to lower their fares when competitors raise their fares and all airlines increase their fares as the departure date gets closer. K.
Obeng (2008) developed a conceptual model to analyze airlines fares in a medium size market using on-line daily information fares, plane and flight characteristics, and trip characteristics collecting data using the ORBITZ Internet search engine. His results show large differences in fares among the airlines, large variation in daily fares offered, and fare differentiation. Fare dispersion can originate from price discrimination (airlines that segment their customers and charge each segment different fares), Edgeworth cycles (period of time or seasonal), peak load pricing (airport charges different cost according to peak operation times) and cost differentials (different airline and airport costs). Competitive routes shows more price discrimination than routes operated by a single airline as a monopoly and more fare dispersion is found on those routes (Giaume and Guillou, 2004). To measure fare dispersion, different indices can be used such as the Gini Coefficient, the Herfindhol Index, the Atkinson Index, the Entropy Index (Hayes and Ross, 1998) and the coefficient of variation, which allows comparison of the variability of prices on different routes (Giaume and Guillou, 2004).

These results show how difficult it is to estimate airfares. K. Obeng (Obeng, 2008) is the only one that considered airlines and airports costs as factors to estimate fares. For this reason, this paper focus on airlines operational costs and airport charges, on a certain route by a specific airline, based on the analysis of the existing relationship between real fares data and airline operations costs and airports charges parameters. This model has been set up by using data available in the United States Domestic Airline Fares Consumer Reports 2005, 2006, 2007 and 2008. Each quarter of the year the U.S. Department of Transportation Office of Aviation Analysis releases a Domestic Airline Fares Consumer Report that includes information of 1,000 largest city-pair markets in the 48 contiguous states. The reports include the number of one-way passenger trips per day, the nonstop distance, average fares per route, the average market share, the airlines with the largest market share and the lowest average fare, and their respective market shares for twenty eight United States (US) airlines, one hundred and thirty three US cities (more than one hundred thirty three airports) and one thousand routes per year. These databases have been used to study airfare pricing (Vowles, 2006), concluding that pricing in hub-to-hub markets is affected by different variables such as route type, presence and type of low fare carriers, and the competition in hub-to-hub markets. Carmona and Lodewijks (2010) use a similar mathematical model to study the LCC’s and FSC’s effects in the US air transport market.

In this study, the airlines operations costs per route estimated have been validated using real airlines operations costs data. This database includes detailed airline reports and queries containing a variety of airline operation costs such as fuel and oil, maintenance, navigation fees, landing fees, ticket and sales, aircraft operations costs and administration.

The aim of this paper is to analyze and develop a mathematical model that estimates airlines operations costs and airport charges per route to help airlines trying to open new routes and for data generation for other purpose such as transport modeling and simulation modeling. Chapter 2 concern the airlines operations costs and airport charges model design. Chapter 3 shows the model results and the correlations, sum of square errors and the sum of square percentage errors analyses between the domestic airlines fares and the fares estimated using the model for the city-pair market.
A mathematical model to predict the US Airlines operation costs and airports charges database. Chapter 4 shows the routes fares estimation for a future year using the model results in year $t-1$ and the inflation rate effect equation. The results are compared with the real fares in year $t$ and also with the model results for year $t$. Chapter 5 shows the Airline operational costs (AOC) model results validation comparing with real airlines operational costs database. Finally, chapter 6 is a conclusion of this paper.

2 Airlines operations costs and airport charges model

To be able to calculate airline operations costs and airport charges, the proposed model use these variables as parameters to estimates airlines routes fares. So the model is actually trying to estimate route fares by using airline operations costs and airport charges. A multi-regression analysis determine the relation between distance, airline operational costs and airport charges between origin and destination and market fares.

From the analyses of the United States Domestic Airline Fares Consumer Reports data, it turns out that distance between origin and destination is the major factor that affects airlines operational costs and airports charges per route, Figure 1. The figure shows that as distance increases the unite price per mile decreases and less dispersion between data exist. Figure 2 shows data for LCC’s during 2005 extracted from Figure 1. Figure 1 and Figure 2 show that high dispersion exists for short-haul markets (less than 750 miles distance between origin and destination cities) because as distance increase the unite price per mile decreases. This result confirms that it is very difficult to develop a long-haul low-cost market. Short routes offer major opportunities to achieve cost advantages. In medium or long-haul operations the duration of flights and passenger minimum quality service requirements reduce the possibilities to minimize airline operation costs.

![Figure 1: Fare/Distance VS Distance model 2005](image-url)
Revenue management is used by airlines to implement pricing policies. Low-cost carriers use an algorithm to compute posted fares as function of the itinerary, departure time, date, time of purchase before departure, and seat availability on the flight. Full service carriers have different procedures that allow carriers to offer multiple fares on a single flight (Alderighi, 2009). For these reasons, fares show high dispersion in same distance routes, but the aircraft operational costs should be similar for same aircraft types. On the other hand, airport charges depend more on the existing relationship between airlines and airports.
Some airline and aircrafts operational costs are: flight crew salaries and expenses, fuel and oil, airport and en-route charges, aircraft insurance, rental/lease of flight equipment/crews, engineering staff costs, spare parts consumed, maintenance administration, flight equipment, ground equipment and property, crew training, ground staff, buildings, equipment, transport, handling fees, cabin crew salaries and expenses, passenger insurance block hours, aircraft performance, aircraft size, mile flown, aircraft design and age (Doganis, 2002, Radnoti, 2001). Figure 3, shows the airline operational costs that can be estimated by the AOC model.

### 2.2 Airports charges factors

Variables such as geographical location, tourism, population, catchment area, accessibility and available capacity of the airports determine the advantages and disadvantages that airports have over each other. These affect the contractual conditions between an airport and an airline. In other words, these variables determine whether an airport has the power to charge airlines or whether it has to finance routes in order to attract more passengers and to increase the economy of the city. It has a direct effect on the airline fares because each city has an economic factor $C_i$ that increase or decrease fares.

![Airport cost factors (ACF)](image)

**Figure 4: Airport cost factors (ACF)**

Airport revenues are classified into aeronautical and non-aeronautical. Aeronautical revenues include only revenues generated via service and facilities related to aircraft operations, passengers and cargo. Non-aeronautical revenues are the ones produced by commercial services and facilities at the airports (Neufville and Odoni, 2003). Thus, airports main goals are to serve more airlines because it represents the opportunity to increase passenger volume and non-aeronautical revenues (Guillen and
Morrison, 2003). Figure 4 shows the airport aeronautical and non-aeronautical revenues according with Neufville and Odoni, 2003.

The most common aeronautical charge is called landing fee, It is the fee that airlines pay to use the airfield (runaway and taxiways) calculated in reference to the maximum take-off or landing aircraft weight, depending on the airport. Terminal area air navigation fee is the airline’s cost for the use of runways and taxi lights, airport radar, instrument landings system, and traffic control. Aircraft parking areas and hangar fees allow them the use of contact and remote apron stands, and sometimes hangar space and it is calculated as a proportion of either the weight or dimension of the aircraft. Airport noise fees directly depend on the time of the day, peaking at night. Passenger fees or terminal service fees cover costs for the use of passenger buildings. Cargo service fees cover the cost of cargo processing facilities and the service executed by the airport, and it is calculated as a fee per ton of freight. Security fees cover costs of security. Ground handling fees are divided into ramp handling fee and traffic handling fee, or passenger handling and cargo handling. En route air navigation fee covers the cost of civil aviation authorities or similar bodies (Neufville and Odoni, 2003).

Non-aeronautical fees are often known as commercial revenues. Concession fees for fuel and oil are charged for the fuel sold to the airline at the airport. Concession fees for commercial activities include small enterprises inside the faculty such as duty-free shops, retail shops, bars, restaurants, banks, currency exchange, etc. Revenues from car parking and car rentals are charged for these facilities at the airport. Rental of airport land, space in buildings, and assorted equipment mainly derive from space rented to airlines for office and passenger “club” lounges, equipment rented to shippers, freight forwarders, advertising in space, etc. Other fees are charged for airport tours, admissions, etc, and some of them are derived from provisions of engineering services and reimbursable utilities by the airport operator to airport users. Finally, non-airport revenues refer to the consulting, educational and training service to other airports (Neufville and Odoni, 2003).

2.3 Airline Operational Costs (AOC) and Airport Charges Factors (ACF) Model.

A fare estimation model can be divided in two different costs models, airlines operational costs (AOC) model and airport charges factors (ACF) model and the profit or lost, as it is shown in equation 1. The objective of this paper, as it has been mentioned, is not to estimate fares, but the model uses fares database to estimate airlines operational costs and airport charge factors. The profit or lost are the errors and can be estimated using other kind of variables such as airline competition, economic and social parameters affecting the US domestic air transportation system.

To minimize operation costs, drop fares and maximize profits, airlines apply different strategies. Those strategies have a direct impact on the airlines operations costs and they are an important reason why airlines have different operational costs. The airline operation costs can be divided into Direct Operation Costs (DOC) and Indirect Operation Costs (IOC) (Equation 2). IOC cost per passenger is equal to the unit handling costs per passenger (Hsu and Wen 2003).
A mathematical model to predict the US Airlines operation costs and airports charges

\[ FARE_{ija} = AOC_{ija} + ACF_{ija} \pm (profit \ or \ lost)_{ija} \tag{1} \]

Where:

- \( AOC \) = Airline operations costs per route per passenger (AOC model) [\$]
- \( ACF \) = Airports cost factors per passenger [\$]
- \( i \) = Origin airport [-]
- \( j \) = Destination airport [-]
- \( a \) = airline \(^1\)

\[ AOC_{ija} = DOC_{ija} + IOC_{ija} \tag{2} \]

where:

- \( DOC \) = Direct operational costs per route per passenger [\$]
- \( IOC \) = Indirect operational costs per route per passenger [\$]

The sum of the operational costs in Figure 3 is the total AOC costs (Equation 3).

\[ AOC_{ija} = SMA_{ija} + SF_{ija} + \cdots + DEP_{ija} \tag{3} \]

To estimate airlines operations costs on certain routes, airlines operations costs factors \( f_a \) are multiply by the distance routes and a constant value \( A \). These variables have been estimated by least square method using real fares data and validated by comparing directly with real airlines operations costs factors, AviationDB database. Thus, the AOC model is defined as:

\[ AOC_{ija} = f_{AOCa}^D \beta A \alpha \tag{4} \]

Where:

- \( f_{AOCa} \) = Airlines operations costs factors per route per passenger [\$/mi]
- \( D \) = Distance [mi]
- \( A \) = Constant [\$]
- \( \beta \) = constant [-]
- \( \alpha \) = constant [-]

Using equation 4, it is possible to calculate each airline operations costs per route per passenger. Equation 5 shows that the AOC model can be used to estimate DOC and IOC costs. Equation 6 shows how to calculate each airline operations costs per route per passenger. Using the value of the variables \( A \), \( D \) and \( \alpha \), and not using the airline operation costs factor \( f_{AOCa} \), estimated by solving equation 4. \( f_{AOCa} \) has to be changed

---

\(^1\) American Airlines (AA), Aloha Airlines (AQ), Alaska Airlines (AS), Jetblue Airways (B6), Continental Airlines (CO), Cathay Pacific Airways (CX), Independence Air/Discovery Airways (DH), Delta Airlines (DL), Boston-Maine Airways DBA Pan Am (E9), Frontier Airlines (F9), Airtran Airways (FL), Allegiant Air (G4), American West Airlines (HP), Spirit Airlines (NK), Northwest Airlines (NW), Skywest Airlines (OO), Pan American Airways (PN), Horizon Air (QX), Sun County Airlines (SY), American Trans Air (TZ), USA 3000 Airlines (US), United Airlines (UA), US Airways (US), Southwest Airlines (WN), Casino Express (XP), Mesa Airlines (YV), Midwest Express Airlines (YX)
for the sum of the average airlines operations costs by dividing the real total amount of airlines expenses, per each different airline operation costs, and the total miles flown by the airline during the year, data available in the Aviationdb database.

\[ AOC_{ija} = AD_{ij} \left( \beta_{DOCa} f_{DOCa} + \beta_{IOCa} f_{IOCa} \right) = AD_{ij} \beta_{DOCa} f_{DOCa} + AD_{ij} \beta_{IOCa} f_{IOCa} \]  \hspace{1cm} (5)

Where:

\[ f_{DOCa} = \text{DOC airlines operations costs factors per route per passenger} \] [\$/mi]

\[ f_{IOCa} = \text{IOC airlines operations costs factors per route per passenger} \] [\$/mi]

\[ \beta_{DOCa} = \text{constant} \] [-]

\[ \beta_{IOCa} = \text{constant} \] [-]

\[ X_{ija} = AD_{ij} f_{xa} \] \hspace{1cm} (6)

Where:

\[ f_{xa} = \text{Airlines operations costs factors per route per passenger} \] [\$/mi]

\[ X = \text{Airline operation cost, X can be any one of all, Figure 3} \] [\$/mi]

Airport charge factor differ for each city. It is very complicated to find information regarding costs and revenues that each airport has for routes to other airports. Most of this information is confidential. For this reason, in this study city factor costs \((C)\) have been introduced by adding the airport charges factors (ACF model, equation 7) to the AOC model. Thus,

\[ ACM_{ij} = \text{Airport charges per passenger} \] [$]

\[ C_m = \text{origin airport cost factor per passenger} \] [$]

\[ C_k = \text{destination airport cost factor per passenger} \] [$]

\[ m, k \text{ = represent different cities} \] [-]

\[ \gamma = \text{constant} \] [-]

Finally, the model to estimate the total cost per route per passenger proposed in this paper is:

\[ TC_{ija} = AF_{AOCa} D_{ija} + C_m^\gamma + C_k^\gamma \] \hspace{1cm} (8)

Where:

\[ TC_{ija} = \text{Total cost per route per passenger per airline} \] [$]

The above model is to estimate airlines operational costs and airports cost factors per route per passenger.
3 Results

Figure 5 shows the factors cost for twenty one airlines. As it was expected, FSC operation costs are higher than low cost carriers. Frontier Airlines (F9) is the only low cost carrier that shows very high operation costs, as much as the FSCs during 2005, 2006 and 2007. In 2008 this carrier reduced its operation costs to the level of the other LCCs. Skybus Airline (SX) appears just during 2007 and has the lowest operation costs. Airline operations costs were very expensive during 2007; follow by 2006, showing a considerable reduction of airline operations costs during 2008.

The correlations between airline operation factors \( (f) \) are very high between consecutive years. This means that airline operation costs change very similar. Table 1 shows over 93% between 2005 and 2007. The correlation between 2007 and 2008 is 83% because as it can be noticed on Figure 5, airlines reduced their operations costs factors \( (f's) \) during that year.

Table 1: Correlation values between 2005, 2006, 2007 and 2008 airlines costs factors \( (f's) \)

<table>
<thead>
<tr>
<th>Year</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.95</td>
<td>0.82</td>
<td>0.68</td>
</tr>
<tr>
<td>2006</td>
<td>-----</td>
<td>0.94</td>
<td>0.79</td>
</tr>
<tr>
<td>2007</td>
<td>-----</td>
<td>-----</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Figure 6 shows the city costs factor \( (C) \) for all the United States airports. When an airport cost factor \( (C_m) \) is negative, means that those airports charge less than the market average charges per passenger. On the other hand, airports with positive charges factor are more expensive than the market average charges.

The airports with the lowest airports cost factors are in tourism places and these results are similar to the results showed by Vowles, T.M. (2000). Vowles, T.M.
(2000) include a resort variable showing that cities with economies based on tourism are more likely to have cheap fares. Florida is the state with lowest cost factor for all their cities.

![Figure 6: Airport city-pair market airport operation cost factor (C’s)](image)

The relationships between airport and airlines determine the costs that airlines have to pay to airports for their operations. Las Vegas has a very low cost factor because South West Airline has a big presence at this airport and it can be identified as a South West Airline hub. South West brings millions of passengers each year to Las Vegas. This airport can minimize the costs to the airline and drop fares because its revenues can be raised by other sources. The city main business is tourism and low fares maintain high volume of passengers during the year. The same thing happens to the cities in Florida. Florida has a tourism economy and hotel companies can even finance flights just because they know visitors spend enough money in tourism activities raising the GDP of the cities.

Aspen and Cincinnati have the most expensive airports costs factors over 90 usd. The majority of the airports have cost between 50 and -50 usd. Hub and multi-hub airports have low cost factors. Some of them have positive and negative factors depending on the year such as Los Angeles Airport (LAX). The majority of them have a negative cost factor what means they want to serve more airlines and bring more passengers offering competitive airport charges. In the case of New York, Newark is the most expensive of its airports with a positive cost factor whilst John F. Kennedy and LaGuardia have negative cost factor. Chicago O’Hare and Chicago Midway have negative cost factor, both airports have very low charges for airline operations, it may be caused by the competition between both airports to serve the same routes and because of the high presence of low cost airlines operating routes served by Chicago Midway Airport.
Most of the regional and secondary airports have a positive cost factor. Cities with large and hub airports depend on the existing level of competition between airlines and airports.

The correlations between airports charges ($C$'s) are high between consecutive years (Table 2). This means that airport operation costs change similar from year to year but not as much as it does in the case of airlines operations costs.

<table>
<thead>
<tr>
<th>Year</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.83</td>
<td>------</td>
<td>0.72</td>
<td>0.62</td>
</tr>
<tr>
<td>2006</td>
<td>-----</td>
<td>0.89</td>
<td>------</td>
<td>0.78</td>
</tr>
<tr>
<td>2007</td>
<td>-----</td>
<td>-----</td>
<td>------</td>
<td>0.78</td>
</tr>
</tbody>
</table>

The estimation of the airport cost factors ($C$'s) is very important to understand and have a good idea about the behavior of the air transportation system, and to eliminate fare dispersion as much as possible to be able to estimate fares close to the reality.

### 4 Forecasting airlines operations costs and airports charges

To evaluate and compare the real value of fares after a given period of time, economic agents are often used to estimate the future. Inflation is a raise in the general level of prices of goods and services in an economy over a period of time, i.e. one year to another. A measure of price inflation is the inflation rate, the annualized percentage change in a general price index. Table 3 shows inflation rates for 2005, 2006, 2007 and 2008.

<table>
<thead>
<tr>
<th>Year</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation rate (%)</td>
<td>3.39</td>
<td>3.24</td>
<td>2.85</td>
<td>3.85</td>
</tr>
</tbody>
</table>

Table 4 shows the models results forecasted for next year $t$ with the model results in year $t-1$ using equation 8, and using equation 9 to compound charges in year $t$ value of the money. The results show that data for all the years under study lay down in a representative model. Airlines operations costs and airports charges forecasted values for the four years show good correlations between real data in year $t$ and the forecast data in year $t$ using $t-1$ parameters values and the inflation rates. The correlation results show a very high correlation between AOC and ACF costs estimated in year $t$ using equation 8 and the AOC and ACF costs estimated in year $t$ using the estimated parameters values, using equation 8, in $t-1$ and the inflation rate. The results show that the inflation can be used to forecast airlines operations costs and airports charges factors per route per passenger as it is shown in Figure 7.

$$FV=PV*(1+i_1)*(1+i_2)*(1+i_3)*...*(1+i_t)$$  \hspace{1cm} (9)
Where:
\(FV\) = Fare future value
\(PV\) = Fare present value
\(i\) = inflation rate
\(t\) = year

Table 4: Relation values, Sum of sqrt errors and percentage errors results for airlines operations costs and cities charges forecast for year \(t\) using the model results for year \(t-1\) and the inflation rate

<table>
<thead>
<tr>
<th>Year</th>
<th>Year</th>
<th>Fare(<em>{est}) (t-1) VS Fare(</em>{real}) (t)</th>
<th>Fare(<em>{est}) (t-1) VS Fare(</em>{real}) (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(T)</td>
<td>(R)</td>
</tr>
<tr>
<td>2005</td>
<td>2006</td>
<td>0.81</td>
<td>5,046,147</td>
</tr>
<tr>
<td>2006</td>
<td>2007</td>
<td>0.85</td>
<td>3,323,815</td>
</tr>
<tr>
<td>2007</td>
<td>2008</td>
<td>0.82</td>
<td>1,883,199</td>
</tr>
</tbody>
</table>

Table 5 shows the percentage error results, for two different ranges (±10%, ±20%), for the analyses between real data and the \(t\) data estimations using \(t-1\) data and the inflation factors. Approximately 50% of the estimations are inside an error range ±10%, for 2006-2007 and 2007-2008 over 80% of the estimations are inside an error range ±20%. In the case of 2005-2006, over 75% of the data has been estimated inside an error range ±20% showing more dispersion as it is shown in Figure 8. Table 5 also shows the percentage error results, for two different ranges (±10%, ±20%), for the analyses between \(t\) forecast using \(t-1\) data and the inflation factor, and the year \(t\) forecast using \(t\) data. The best results are for 2006-2007 forecasting, 94% of the estimations are inside an error range ±10% and 99% of the estimations are inside an error range ±20%. In the case of 2005-2006, the estimations show more dispersion over 76% of the data has been estimated inside an error range ±10% and 95% inside an error range ±20% as it is shown in Figure 9.

Figure 7: Correlation between estimations in 2006 and the estimation in 2007 using the inflation rate
Table 5: Percentage of forecasting interval errors

<table>
<thead>
<tr>
<th>Year</th>
<th>Year</th>
<th>Fare\textsubscript{est} \textsubscript{t-1} VS Fare\textsubscript{real} \textsubscript{t}</th>
<th>Fare\textsubscript{est} \textsubscript{t-1} VS Fare\textsubscript{est} \textsubscript{t}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>±10% error routes inside</td>
<td>±20% error routes inside</td>
</tr>
<tr>
<td>2005</td>
<td>2006</td>
<td>0.44 1,622 0.75 2,791</td>
<td>0.76 3,050 0.95 3,818</td>
</tr>
<tr>
<td>2006</td>
<td>2007</td>
<td>0.50 1,889 0.82 3,090</td>
<td>0.94 3,770 0.99 3,951</td>
</tr>
<tr>
<td>2007</td>
<td>2008</td>
<td>0.46 645 0.80 1,117</td>
<td>0.89 3,563 0.99 3,979</td>
</tr>
</tbody>
</table>

Figure 8: Error distribution forecasting real data in year \(t\) by using the model results in year \(t-1\)

Figure 9: Error distribution forecasting real data in year \(t\) by using the model results in year \(t-1\) and the inflation rate

5 Results validation

The airlines operations costs factors estimated using equation 4 and the airlines operations costs calculated using real airlines operation costs data are not expected to be exactly the same because other factors are affecting the airlines operations costs, such as wind flows and airports altitude, what means two routes operated by the same
carrier with equal travel time distance do not have the same airline operation cost. Equation 4, 5 and 6 have been used to calculate airline operations costs using the factors calculated by the total airlines operations costs in all their routes per miles flown by the airline per day, instead of using the estimated $f_{AOCa}$ values. Figure 10 shows the correlation between the airlines operations costs estimated using the AOC model and the airlines operations costs calculated using the total operations costs per day and the airlines total travel distance (mi) per day for the complete airlines domestic market using the Aviationdb database. The results confirm that the AOC model, equations 4, 5 and 6 calculates the airlines operations costs, with very high accuracy, by estimating airlines operations costs using fares data per airline route by minimizing the sum of square errors between real fares and airlines operations costs estimated by the model. Table 6 and Figure 11 show the correlation results, the sum of square errors and the sum of square of the percentage errors between the airlines operations costs estimated by the AOC model, equation 4, and the airlines operations costs factors calculated with real airlines operations costs data.

Table 6: Correlation, sum of square errors and sum of square percentage errors between real airlines operations costs and the airlines operations costs estimated by the model equation 4

<table>
<thead>
<tr>
<th>Year</th>
<th>$R$</th>
<th>$R^2$</th>
<th>Sum of sqrt errors</th>
<th>Sum of sqrt % errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.99</td>
<td>0.98</td>
<td>101,905</td>
<td>3.49</td>
</tr>
<tr>
<td>2006</td>
<td>1.00</td>
<td>1.00</td>
<td>21,331</td>
<td>0.75</td>
</tr>
<tr>
<td>2007</td>
<td>1.00</td>
<td>1.00</td>
<td>16,766</td>
<td>0.75</td>
</tr>
<tr>
<td>2008</td>
<td>0.99</td>
<td>0.98</td>
<td>168,541</td>
<td>4.81</td>
</tr>
</tbody>
</table>

Figure 10: Correlation between airlines operations costs (AOC) estimated using the mathematical model ($f's$) and the airlines operations costs (AOC) for the complete industry i.e. 2007
A mathematical model to predict the US Airlines operation costs and airports charges

Figure 11: Error distribution between airlines operations costs estimated using the mathematical model ($f$'s) and the airlines operations costs per mile for the complete industry

The airline operation cost estimation model (AOC), equation 4, shows a very good correlation results for all the years under study. Table 7 shows how accurate the AOC model is when predicting airlines operations costs per route. The results have found that the airlines operations costs model, equation 4, estimates all the airlines operations costs inside ±20% and from the 4,000 routes under study the model estimates over 98% inside ±10% percentage error what demonstrated that the model is a good tool to estimate airlines operations costs per route.

Table 7: Percentage of data interval errors

<table>
<thead>
<tr>
<th>Year</th>
<th>±10% errors</th>
<th>Routes inside ±10%</th>
<th>±20% errors</th>
<th>Routes inside ±20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.98</td>
<td>3,936</td>
<td>1.00</td>
<td>4,000</td>
</tr>
<tr>
<td>2006</td>
<td>1.00</td>
<td>3,997</td>
<td>1.00</td>
<td>4,000</td>
</tr>
<tr>
<td>2007</td>
<td>1.00</td>
<td>4,000</td>
<td>1.00</td>
<td>4,000</td>
</tr>
<tr>
<td>2008</td>
<td>0.99</td>
<td>3,970</td>
<td>1.00</td>
<td>3,987</td>
</tr>
</tbody>
</table>

6 Conclusions

The results show that airlines operations costs and airport charges factors can be estimated by the proposed model with high accuracy. The study also proves that airlines operation costs and airport charges are very important parameters to estimate fares, because fares are higly determined by airlines operations costs and airport charges. In this study, the profit or lost were not study, but the authors of this paper have developed a multi-regression fare estimation model to estimates fares by considering airlines strategies, social, economic and competition factors.
The forecasting results show that the AOC model can estimate airlines operations costs for year \( t \) using the model results in year \( t-1 \) and the inflation rate. The correlation results are very high between model estimations in year \( t \) and the model estimation results in year \( t-1 \) and forecasted using the inflation rate. The relation between airline operations costs for year \( t \) using the model results and the inflation rate showed for year \( t-1 \) are over 96% with very low sum of square errors and sum of square percentage errors as it is shown in Figure 7. The model forecasts over 99%, from the total 4,000 routes under study, inside \( \pm 20\% \) percentage error and for year 2007 over 94% inside \( \pm 10\% \) percentage error showing 89% and 76% for 2008 and 2005 respectively Figure 9.

The Airlines operations costs factors estimated by the model are not expected to be the same for routes operated by the same airline with same travel distances because other factors are affecting the airlines operations costs such as airports altitude and wind flows. The airlines operations costs per route estimated have been validated using real airlines operations costs. This database includes detailed airline reports and queries containing a variety of airline operation costs such as fuel and oil, maintenance, navigation fees, landing fees, ticket and sales, aircraft operations costs and administration. Figure 10 and 11 compares the AOC model against real airlines operations costs data, showing correlation results over 99%. From the 4,000 routes under study, the model estimates over 3,970 routes inside \( \pm 10\% \) percentage error and 4,000 inside \( \pm 20\% \) percentage error. The results confirm that the AOC model estimates airlines operations costs very accurate what means the AOC model can be used as a tool to estimate and compare the average airlines operations costs between airlines and different airlines business models.

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