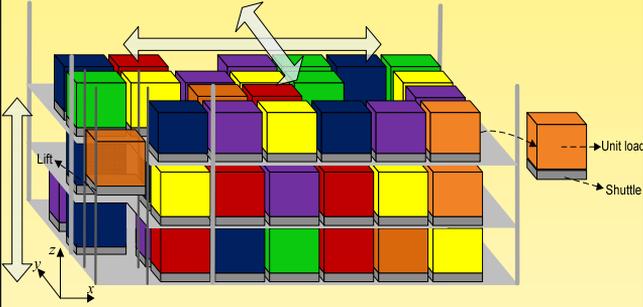
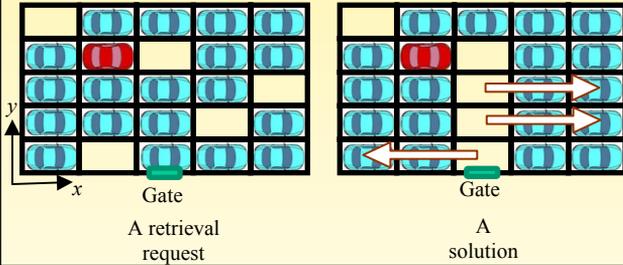


### Introduction

#### • Puzzle-based compact storage system



#### • Work mechanism



### Objective

• **Research question:** How to optimize dimensions of a puzzle-based storage system, leading to the minimum response time in a random storage policy.

#### • Assumptions:

1. The system capacity is a known positive constant.
2. A random storage policy is assumed.

• **Theorem.** The minimum retrieval time of a random unit load stored at location  $(X, Y, Z)$ , can be estimated by the following equation:

$$RT(X, Y, Z) = \max\{X+Y, Z\} + Z, \quad (1)$$

if the utilization of the system does not exceed  $(V' - \max\{L, W\})/V'$ , where  $V'$ ,  $L$ , and  $W$  represent the capacity of the system in number of storage locations, the number of columns in each level, and the number of rows in each level, respectively.

A random retrieval location can be denoted by  $(X, Y, Z)$  where  $X$ ,  $Y$  and  $Z$  refer to coordinates in x-, y- and z-directions respectively.

### Mathematical model

$$\text{Min } ERT, \quad (2)$$

subject to:

$$l \times w \times h = V, \quad (3)$$

decision variables:  $l > 0, w > 0, h > 0,$

where  $l$ ,  $w$ , and  $h$  (length, width, height of the system) are the decision variables. All the dimensions are expressed in time units.  $V$  represents the volume of the system in cubic time unit. Constraint (3) makes sure the given capacity is achieved.

### Methodology

• The model is non-linear and mixed integer; however, we can optimally solve it by splitting it into several solvable sub-models

• In order to solve the model we have to derive ERT in Equation (2). The expected retrieval time for any puzzle-based system with a given capacity can be calculated as follows:

$$ERT = \int_{t=0}^{\max\{w+l, h\}+h} f(t) dt \quad (4)$$

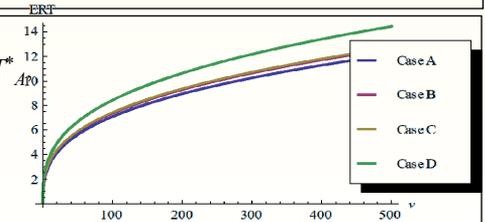
where,  $t$  represents the retrieval time for any retrieval location.  $f(t)$  represents the probability density function of retrieval time  $t$ .

• Calculation of ERT can be done into four different complementary cases, each referring to a specific configuration of the system:

**Case A:**  $h \leq w$ , **case B:**  $w < h \leq l$ , **case C:**  $l < h \leq l+w$ , **case D:**  $l+w < h$ .

### Findings and conclusions

The following figure give  $ERT^*$ ,  $ERT_B^*$ ,  $ERT_C^*$  and  $ERT_D^*$  as a function of  $V$ .



The solution of case A ( $h \leq w$ ) gives the minimal ERT for the Model.

$$h^*(V) = 0.874461V^{1/3} \quad (5)$$

$$l^*(V) = w^*(V) = 1.069374V^{1/3} \quad (6)$$