Using results from multi-objective optimization as decision support information in multimodal passenger transportation network design

**TRAIL Research School, November 2014** 

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# Contents

	Abstract	3
1	Introduction	4
2	Problem definition	5
2.1	Mathematical formulation	6
2.1.1 2.1.2 2.1.3 2.1.4	Network and demand definition Optimization problem Multi-objective optimization: Pareto optimality Lower level model	6 7
2.2	Study area	7
2.2.1 2.2.2	Objective functions Decision variables	
2.3	Solution method	9
3	Results	9
3.1	Traditional visualisation	9
3.2	More dimensional scatter plots	11
3.3	Stepwise reduction of the number of solutions	13
3.3.1 3.3.2	Using values of objective values Using values of decision variables	
3.4	Choosing compromise solutions	15
3.4.1	Defining priorities among objectives	16
4	Conclusions	17
Ackn	owledgement(s)	
Refer		

## Abstract

Given a range of traffic related sustainability problems, the question for policy makers arises what measures should be taken to reach their objectives as much as possible. Multi-objective optimization is useful to support these decisions, because it results in an overview of possibly optimal solutions. This Pareto set can be very large, especially if more than two (mainly opposed) objectives are involved. This is also the case when optimizing infrastructure planning in a multimodal passenger transportation network, with accessibility, use of urban space by parking, operating deficit and climate impact as objectives. Methods are presented to identify promising solutions from the Pareto set. This involves adding post-optimization constraints to objective functions values, selecting certain decision variables (i.e. measures to be taken) based on political preferences and looking for the best compromise solutions. These methods make the Pareto set more useful as decision support information since they demonstrate the next step in multi-objective option prioritization.

# Keywords

Multimodal passenger transportation networks, multi-objective optimization, multicriteria analysis, decision support, genetic algorithm

## **1** Introduction

Highly urbanized regions in the world nowadays face well known sustainability problems in the traffic system, like congestion, use of scarce space in cities by parking of vehicles and the emission of greenhouse gases. A shift from car to public transport (PT) modes is likely to alleviate these problems. Optimal extension of the PT network contributes to this shift by enhancing the quality of trips made by PT. However, investments in PT infrastructure require large financial recourses. To take more advantage out of the existing PT infrastructure, facilitating an easy transfer from private modes (bicycle and car) to PT modes (bus, tram, metro, train) may stimulate the use of PT, without the need for big investments in large infrastructural developments. This transfer can be eased by network developments that enable multimodal trips (that combine private and public modes), like opening new park and ride facilities, opening of new train stations or opening new or changing existing PT service lines.

When such transportation network developments are planned, the decision is often based on an evaluation of a few pre-defined scenarios. The composition of the scenarios is usually based on expert judgment, and the assessment usually is executed using multi criteria analysis. However, the scenario that is selected as the best may not be the best overall: it can very well be that this scenario can be improved. Therefore another method is to optimize the network (known as the network design problem, NDP), for example maximizing the accessibility subject to constraints on externalities, such as an emission reduction target or a budget constraint. This results in one optimal network solution. However, it does not provide insight in the dependencies between objectives, i.e. the extent to which the objectives are opposed or aligned and no information is provided on the possibilities to improve the network further if the budget is slightly increased. Another common method is to combine a set of objectives using a weighted sum, where the weights represent the compensation principle between the objectives. However, setting these weights is not trivial: if these are determined in advance, uncertainty concerning these weighting factors is not incorporated and the sensitivity of the outcome to these factors is not known in advance. For these reasons in this paper another approach is adopted, a so-called multi objective network optimization, that enables us to identify trade-offs between objectives (see also Coello Coello (2006)). Still this is a simplification of real world decision making, where even a specific aspect of the problem (i.e. sustainability) may be operationalized in several ways. However, when the objectives are defined in cooperation with the decision makers, the most important aspects are explicitly taken into account.

NDPs have received a lot of attention in the literature, in many different versions. One subclass of problems is the transit network design problem, which has been studied in various ways, as reviewed by Guihaire and Hao (2008). This includes greedy algorithms, evolutionary algorithms and design meetings involving expert judgments. Another subclass of problems is the unimodal road network design problem, which has also been widely studied (Yang and Bell 1998).

As is also identified in the more recent review by Farahani et al. (2013), a more specific class is the combination of public and private modes: the multimodal network design problem. In this area, previous studies addressed several decision problems and, accordingly, several ways to model the choices that travellers have in a multimodal network, of which the most relevant studies are mentioned here. Miandoabchi et al. (2012) design road link capacity and bus routes using two types of evolutionary metaheuristics, where the traveller may choose modes between car and bus in the lower level. Hamdouch et al. (2007) describe the problem of

pricing private and public links, with a mode choice for the traveller between car and car combined with metro. García and Marín (2002) focus on park and ride: parking capacity and pricing are decision variables and mode choice between car, metro and park and ride are available in the model. Uchida et al. (2007) focus on development of a probit-based lower level model, where the multimodal network is modelled by a hyper-network, enabling advanced modelling of trips that combine private and public modes. The NDP problem is addressed as well, on a primitive network with the frequency of 2 PT lines as decision variables, for which a local linear approximation is formulated to obtain a solution efficiently in terms of time.

Another more specific class is considering the NDP as a multi-objective problem. For examples, the following relations between objectives can be found in the literature. Travel time and construction costs clearly compete in the case of road network design (Chen et al., 2010). Travel time and CO emissions clearly compete in the case of urban road network design (Cantarella and Vitetta, 2006). Next, Sumalee et al. (2009) include three objectives: social welfare improvement, revenue generation and equity and show that trade-offs exist between all three objectives. Finally, Miandoabchi et al. (2012) maximise user benefits, passenger share of the bus mode, service coverage and minimises average generalized travel cost for a bus mode trip simultaneously, but does not give any insight into how these objectives interact.

To our knowledge Miandoabchi et al. (2012) is the only paper combining multi-objective optimization with a multimodal NDP, considering both new street construction and lane additions / allocations as well as redesign of bus routes. The focus is on the development of the metaheuristics to solve the problem: the performance of a hybrid genetic algorithm and a hybrid clonal selection algorithm are compared, using multiple test networks. In the lower level, car and bus are distinguished as separate modes.

The first contribution of this paper is to apply multi-objective optimization to the multimodal NDP, including multimodal trips in the lower level: the traveller can choose between using a single private or public mode, or using a combination of public and private modes (a mode chain). This enables a focus on multimodal network developments as decision variables, combining park and ride and new train stations (that can also be reached by bicycle) with frequency setting of PT lines. This optimization problem is described in section 2.1. The optimization framework is applied on a real world network (section 2.2). The usefulness of the method is illustrated by a clear presentation of the information that is provided by the Pareto optimal set, in the form of trade-offs between objectives (section 3.1 and 3.2). As a second contribution, we show the next step in multi-objective driven option prioritization, by presenting a stepwise reduction procedure that enables decision makers to choose one final solution for implementation (section 3.3), and a method to choose a final solution by interactively setting weights (section 3.4). In section 4 of the paper we draw conclusions.

# 2 **Problem definition**

The transportation network design problem is often solved as a bi-level optimization problem (Farahani et al., 2013), to correctly incorporate the reaction of the transportation system users to network changes, as is argued by dell'Olio et al. (2006) and Tahmasseby (2009), see Figure 1. The upper level represents the behaviour of the network authority, optimizing system objectives. In the lower level the travellers minimize their own generalized costs (e.g. travel time, cost), by making individually optimal choices in the multimodal network, considering

variety in travel preferences among travellers. The network design in the upper level interacts with the behaviour of the travellers in the network: the lower level. The lower level is a constraint for the upper level problem, since the upper level cannot dictate the behaviour of the users in the lower level. Any network design the network authority chooses results in a network state (e.g. travel times and flows), from which the system objectives can be derived. The bi-level linear programming problem is already NP-hard (Gao et al., 2005), so any problem of this type is NP-hard. Therefore, heuristics are needed to solve the bi-level NDP for larger networks. The huge number of feasible solutions and the non-convexity of the objective function necessarily requires the adoption of metaheuristic algorithms (D'Acierno et al., 2014).

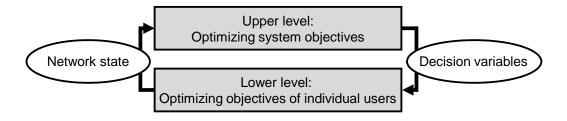


Figure 1: The bi-level optimization problem

### 2.1 Mathematical formulation

### 2.1.1 Network and demand definition

The multimodal transportation network is defined as a directed graph G, consisting of node set N, link set A, a line set L and a stop set U. For each link one or more modes are defined that can traverse that link with a certain speed and capacity: the link characteristics  $C_a$ . Transportation zones and act as origins R and destinations S and are subsets of N. Total fixed transportation demand q is stored in a matrix with size  $|R| \times |S|$ . Furthermore, transit service lines L are defined as ordered subsets  $A_l$  within A and can be stop services or express services. PT flows can only traverse transit service lines. Transit stations or stops U are defined as a subset within N. Consequently, a line l traverses several stops. The travel time between two stops and the frequency of a transit service line l are line characteristics  $C_l$ . Access / egress modes and PT are only connected through these stops. Whether a line calls at a stop u or not, is indicated by stop characteristics  $C_u$ . All together, the transportation network is defined by G(N, A, L, U), where A, L and S are further specified by  $C_a$ ,  $C_l$  and  $C_u$ .

### 2.1.2 Optimization problem

We define a decision vector  $\underline{y}$  (or a solution), that consists of *V* decision variables:  $\underline{y} = \{y_1, \dots, y_v, \dots, y_v\}$ . *Y* is the set of feasible values for the decision vector  $\underline{y}$  (also called decision space). The objective vector  $\underline{Z}$  (consisting of *W* objective functions,  $\underline{Z} = \{Z_1, \dots, Z_w, \dots, Z_W\}$ ) depends on the value of the decision vector  $\underline{y}$ . Every  $\underline{Z}$  is part of the so called objective space, and in principle  $\underline{Z}$  may be any value in  $\mathbb{R}^W$ , but depending on its meaning, an objective function may be subject to natural bounds. In this paper we will suffice with a formulation that states that the lower level should be in user equilibrium (see Eq. 1 and section 2.1.4). For a more detailed formulation of the optimization problem we refer to (Brands and van Berkum, 2014).

 $\min \underline{Z}(y)$ , subject to

 $\underline{y} \in Y$   $G\left(N, A\left(C_{a}(\underline{y})\right), L\left(C_{l}(\underline{y})\right), S\left(C_{s}(\underline{y})\right)\right)$ satisfies SUE for q

#### 2.1.3 Multi-objective optimization: Pareto optimality

Mathematically, the concept of Pareto optimality is as follows. If we assume two decision vectors  $\underline{y}_i, \underline{y}_i \in Y$ , then  $\underline{y}_i$  is said to strongly dominate  $\underline{y}_i$  iff  $Z_w(\underline{y}_i) < Z_w(\underline{y}_i) \forall w$  (also written as  $\underline{y}_i \prec \underline{y}_i$ ). Additionally,  $\underline{y}_i$  is said to weakly dominate (or cover)  $\underline{y}_{i'}$  iff  $Z_w(\underline{y}_i) \leq Z_w(\underline{y}_i) \forall w$  (also written as  $\underline{y}_i \prec \underline{y}_i$ ). All solutions that are not weakly dominated by another known solution are possibly optimal for the decision maker: these solutions form the Pareto-optimal set *P*.

### 2.1.4 Lower level model

The lower level model calculates the network flows through the multimodal network. Therefore, transportation demand is assumed to be fixed, but mode choice and route choice of travelers is flexible. As defined in the previous section, the decision variables typically involve multimodal trip making, for example a park and ride facility involves combining car and PT to a park and ride trip and a new train station may involve combining bicycle and PT to a bike and ride trip. To correctly take into account these effects, combinations of different access and egress modes are defined in mode chains. These mode chains are seen as separate modes in the mode choice model, which comprises a nested logit model with choice between car and PT in the main nest and choice between mode chains in the PT subnest. More details of this model can be found in (Brands et al., 2014a).

### 2.2 Study area

The case study area covers the Amsterdam Metropolitan Area in The Netherlands (Figure 2). This area has an extensive multimodal network with pedestrian, bicycle, car and transit infrastructure. Transit consists of 586 bus lines, 42 tram and metro lines and 128 train lines, which include local trains, regional trains and intercity trains. Bicycles can be parked at most bus stops and at all train stations. A selection of transit stops facilitates park-and-ride transfers. Origins and destinations are aggregated into 102 transportation zones. Important commercial areas are the city centres of Amsterdam and Haarlem, the business district in the southern part of Amsterdam, the harbour area and airport Schiphol. Other areas are mainly residential, but still small or medium scale commercial activities can be found.

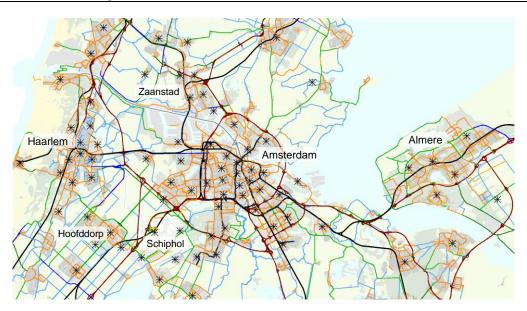


Figure 2: Map of the study area, showing transportation zones, railways, roads.

### **2.2.1** Objective functions

In this paper we consider 4 policy objectives related to sustainability, concerning accessibility, use of urban space by parking, climate impact and costs. These objectives are operationalized as follows (more details can be found in (Brands and van Berkum, 2014)). In our case of a fixed total demand, total travel time (TTT) is used to represent accessibility in  $Z_1$ . The urban space used by parking (USU) is represented by the number of car trips to or from zones that are classified as highly urban, because such a trip requires a parking space that cannot be used for other urban land uses ( $Z_2$ ). These alternative land uses give additional value to property (Luttik, 2000). Operating deficit of the PT system (OpD) is formulated as  $Z_3$ , rather than as a budget constraint, to provide explicit insight in the relation between costs and other objectives. Cost parameters follow from Dutch PT operating practice. CO<sub>2</sub> emissions (CE) represent climate impact ( $Z_4$ ). All 4 objectives are to be minimized.

### 2.2.2 Decision variables

In the network of the study area, 37 decision variables are defined related to transfer facilities or to PT facilities. The decision variables are based on regional policy documents and on interviews with policy makers in the study area. For every potential network development, a decision variable  $y_v$  is defined in advance (see table 1). Opening / closure of train stations, intercity status of train stations and opening / closure of park and ride (P&R) facilities are represented by binary variables. For transit line frequency, a discrete set of choice options is predefined, depending on the expected load for that transit line. Network developments are only included as a candidate location if spatial and capacity constraints are met. For example, a P&R facility is only potentially opened if the corresponding station is served by PT. The characteristics of links, lines and stops that are not candidate locations are fixed at one value. Furthermore, the car and bicycle networks are assumed to be fixed. In this case, the feasible region *Y* contains approximately 7\*10<sup>10</sup> possible decision vectors.

Table 1: Overview of decision variables in the multimodal network design problem.

Decision var- Possible	Represents	Description
iable index v values of $y_v$	real value	Description

Using results from multi-objective optimization as decision support information in multimodal passenger transportation network design 9

1-6	{0,1}	Existence	Opening / closure of train stations
7-9	{0,1}	Existence	Intercity status of train stations
10-16	$\{0,1\}$	Existence	Opening / closure of P&R facilities
17,21,22,24	$\left\{0,\frac{1}{3},\frac{2}{3},1\right\}$	$\{0, 4, 8, 12\}$	Frequency of bus lines
18,19,20,23	$\{0,\frac{1}{3},\frac{2}{3},1\}$	$\{0, 2, 4, 6\}$	Frequency of bus lines
25-32,34,36	$\{0,\frac{1}{3}\}$	$\{0, 2\}$	Frequency of local train lines
33,35	$\left\{0,\frac{1}{3},\frac{2}{3},1\right\}$	$\{0, 2, 4, 6\}$	Frequency of local train lines
37	$\{0,1\}$	Existence	Extension of a tram line

### 2.3 Solution method

The optimization problem defined in Eq. 1 is solved using the evolutionary algorithm  $\varepsilon$ -NSGAII (Kollat and Reed, 2006). This method was earlier shown to outperform the well-known predecessor of the algorithm NSGAII (Deb et al., 2002) when applied to the same case study in Brands et al. (2014b), especially when limited function evaluations are possible due to high computation time. A more detailed description of the algorithm can be found in the same paper.

## 3 **Results**

The optimization algorithm produces a Pareto set as a result. The Pareto set contains information, that is presented in various ways in this section. This helps to better understand the network design problem in a multimodal context and to finally choose one solution from the Pareto set for implementation in a multi-objective option prioritization process. Firstly, the trade-offs between objectives are shown. Examples are shown for pairs of objectives that are mainly opposed to each other and where the relation between the two is less clear. The latter case is clarified by plotting the corresponding values of a third objective. Secondly, a stepwise reduction procedure is presented, enabling decision makers to choose one final solution for implementation, based on additional bound to objective functions as well as selecting specific measures to be implemented.

In total 2384 solutions were calculated during the execution of the algorithm. From these solutions, 210 were Pareto optimal (i.e. non-dominated). This Pareto set is an approximation of the real Pareto set, since it would take too much computation time to calculate all solutions and thus the true Pareto set is not known. For the case study, the calculation of one solution takes approximately 6.5 minutes, implying that execution of the optimization algorithm takes almost two weeks of computation time (using a computer with an Intel® Core<sup>TM</sup> i7 CPU 860 @ 2.8GHz and a 4 GB RAM). Note that the  $\epsilon$ -NSGAII algorithm has possibilities for parallel computing (all solutions in one generation could in theory be computed in parallel), if it would be desirable to reduce computation time.

### 3.1 Traditional visualisation

The scatter plot shown in Figure 2 is a common way to visualize a Pareto set, especially to show trade-offs between objectives. Since only two objectives can be shown per objective, several plots are needed to show all interactions between objectives in a scatterplot matrix (Lotov and Miettinen, 2008). Here, only 2 pairs of 2 objectives are shown. The plots also show solutions which are not Pareto optimal when only the two objectives in the plot are considered, but these solutions are Pareto optimal as a result of the four considered objectives during optimization. The first plot for urban space used (USU) and PT operating deficit

(OpD) shows a clear trade-off. The first improvements in USU are cheap, but further improvements become more expensive in terms of OpD. The second pair of objectives is total travel time (TTT) and  $CO_2$  emissions (CE). These objectives are rather in line with each other: two clusters may be observed, one with lower TTT and lower CE and one with higher values for both objectives. On the other hand, within the lower left cluster also a trade-off between the two objectives can be observed.

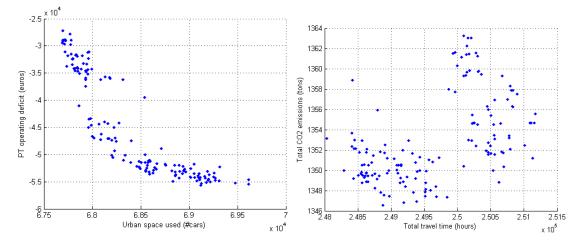


Figure 2: Two scatter plots of the Pareto set: one dot represents one solution in the set, where the corresponding scores for the two objective values can be read on the two axes.

A parallel coordinates plot (see Figure 3) captures all 4 objectives in one plot (as used earlier by Kasprzyk et al. (2013) ). Normalization per objective is required in this type of plots, because the four objectives have different orders of magnitude. The normalized values range from 0 to 1, corresponding to the minimum per objective and the maximum per objective. When interpreting these normalized value, note that one objective may have a large absolute difference between maximum and minimum values and another objective may have a small absolute difference (in case the first objective can be influenced strongly by the decision variables and the latter objective cannot). This is especially relevant when weight factors are used to combine normalized objectives.

In a parallel coordinates plot trade-offs between objectives cannot be observed as directly as in a scatter plot. Using a colour scale to represent one specific objective value improves this. When looking at the colour distribution, it can be observed that high TTT implies low OpD, but usually also high USU. The relation with CE is less clear, but roughly TTT and CE are in line (as was earlier observed in the scatter plot). Putting the objectives in a different order on the horizontal axis and colouring based on different objectives may provide different insights.

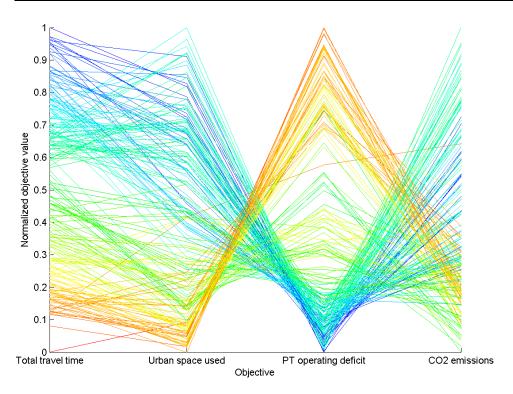


Figure 3: Parallel coordinates plot of the Pareto set: one line represents one solution, where the normalized values of the 4 objective values are plotted in the 4 columns. The lines are coloured using their value for total travel time.

### **3.2** More dimensional scatter plots

By using different colours for the dots in a scatter plot, one additional objective is included in the scatter plot (see Figure 4). This way of visualisation is earlier referred to as decision map (Lotov and Miettinen, 2008). A limited number of categories is defined for the 3<sup>rd</sup> objective which correspond to types of dots as shown in the legend. This type of representation can for instance be used to visualize the effect of introducing a constraint for the 3<sup>rd</sup> objective on the Pareto front of the two objectives at the axes, i.e. its effect on which solutions remain Pareto optimal and their related outcome concerning the two other objectives.

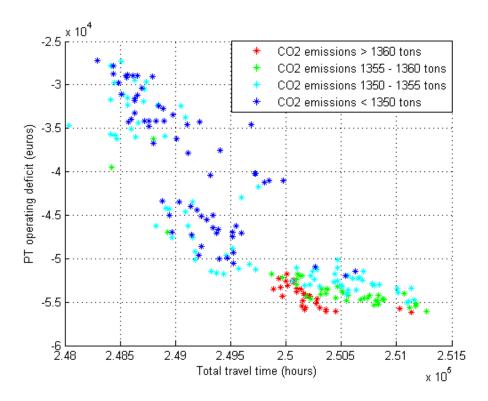


Figure 4: A 2D scatterplot for total travel time and operating deficit, with a distinction between CO<sub>2</sub> emissions categories (also called decision map).

It can be observed that solutions with high CE only occur in an area of the plot with low OpD and medium to high TTT. On the other hand, a very interesting observation is that when only solutions with low CE emissions are considered (i.e. less than 1350 tons), still a large variation exists in scores for TTT and OpD. With other words, when an additional constraint is set for CE, there is still a choice possible between solutions with low TTT, with low OpD or trade-off solutions with intermediate values for both objectives.

A similar plot is shown in Figure 5, where the 3<sup>rd</sup> objective is represented by a continuous colour scale. This could be referred to as a continuous decision map. It contains more detailed information then the decision map based on categories, but it may be more difficult to read. Which of the two to prefer depends on the desired accuracy of the 3<sup>rd</sup> objective. Especially in the centre of the plot interesting trade-off information is found: the solutions closest to the lower left corner of the plot are the best when only TTT and OpD are considered (assuming the corresponding weight factor), but moving a little to the upper right direction results in a large gain in CE at little costs in terms of TTT and OpD.

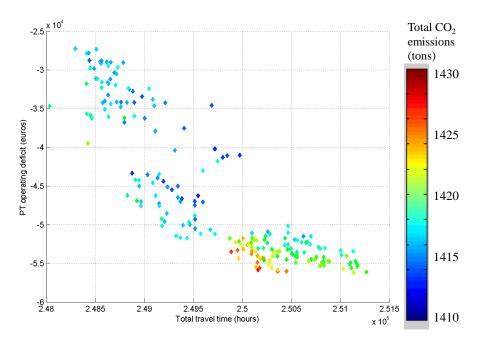


Figure 5: Continuous decision map of the Pareto set

It is even possible to include information on more objectives in such a figure, by using arrows instead of dots in a scatter plot, where the orientation of the arrow represents a fourth objective and the size of the arrow represents a fifth objective (see Kasprzyk et al. (2013) for an example). It is questionable whether all this information may be interpreted together in one figure.

### **3.3** Stepwise reduction of the number of solutions

In this section a stepwise reduction procedure is presented to come to a final decision for implementation based on the Pareto set: either putting additional constraints to objective function values or fixing certain decision variable values, i.e. choosing a measure to implement (for example because it is politically desirable for reasons that are not included in the considered objectives). These two approaches are the result of three interviews that were taken from policy officers that prepare decision making at three different local governments in the Netherlands (municipality of Amsterdam, city region of Amsterdam and province of Overijssel). Note that these two approaches may also be combined to make a selection, but for simplicity that is not done here.

#### 3.3.1 Using values of objective values

Starting from all solutions in the upper left corner of Figure 6, one method (that was suggested by policy officers during the interviews) to stepwise reduce the number of solutions in the Pareto set is to put additional constraints to objectives after optimization. In this example, first an additional constraint to (normalized) PT operating deficit is set such that only solutions with a value lower than 0.4 are included. As a result 137 solutions of the original 210 solutions are left. One more constraint is put to  $CO_2$  emissions: in addition to the constraint to operating deficit, only solutions with a (normalized) value of lower than 0.2 are included. Only 20 solutions are left now in the selection. As can be seen in the lower left corner of the figure, this selection excludes all solutions with very low values for the other

two objectives: putting a bound on the values for  $CO_2$  emissions and PT operating deficit implies a bound for the other two objectives as well. A closer look at the objective PT operating deficit reveals that by the constraint for  $CO_2$  emissions, the best solutions for PT operating deficit are now also excluded. Finally, when the decision maker is satisfied by the values for  $CO_2$  and PT operating deficit that are set now, a logical choice from the remaining solutions is to find the best compromise solution for the other 2 objectives (in this example based on the normalized values).

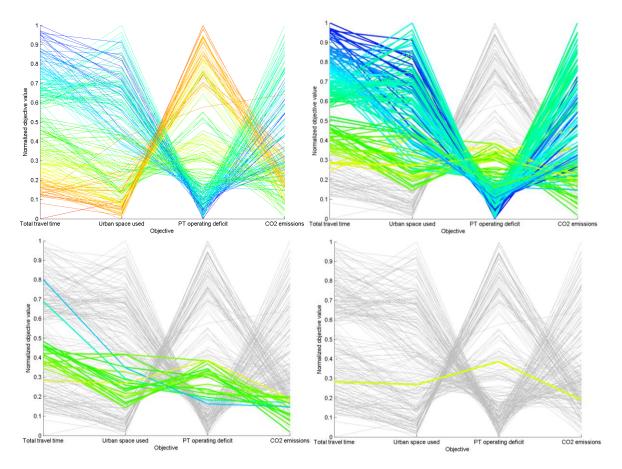


Figure 6: Stepwise reduction from all Pareto solutions to one solution to be implemented by setting bounds to objective values.

#### 3.3.2 Using values of decision variables

Another method that was suggested by policy officers during the interviews to stepwise reduce the number of solutions is selecting certain values for decision variables form the Pareto set. An interactive design process arises, where political preferences (that were not included in the objective definitions) come into play in the search for a final network design to implement. This design of network scenarios after optimization has two advantages over a pre-definition of these scenarios. Firstly, during the choosing process (i.e. in a workshop), the values for objective values are immediately known for each Pareto solution, since the solutions have already been evaluated using the lower level model. Consequently, if a certain choice implies very bad scores for an objective that is considered to be important, the choice is likely to be reconsidered by the decision maker. Secondly, a suboptimal solution (given the four predefined objectives) is never chosen, since all solutions are in the Pareto set (i.e. non-dominated) and therefore all possible as finally optimal solutions.

Using results from multi-objective optimization as decision support information in multimodal passenger transportation network design 15

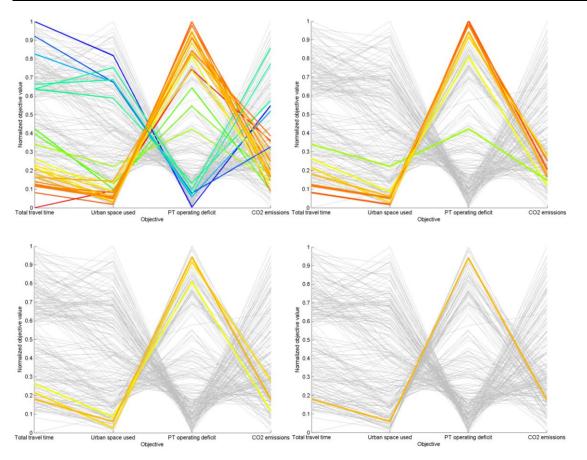


Figure 7: Stepwise reduction to one solution to be implemented by selecting values for decision variables.

An example of such a reduction is shown in Figure 7. First, only solutions that have a frequency of 6 buses per hour on the bus line between Amsterdam Sloterdijk and Schiphol (in Dutch called 'Westtangent') are included. This results in 26 solutions that remain from the 210 Pareto solutions, but for all four objectives, both solutions with low values and with high values are still included. A further selection is for solutions that also include a park-and-ride facility along this new bus line, at Schiphol-North. 7 solutions remain, but now only solutions remain with high values for OpD and low values for the other three objectives (see the upper right plot in Figure 5.14). As a next step selecting the solutions that include the train station in the village of Halfweg (between Amsterdam and Haarlem) result in 3 remaining solutions, which have similar scores for the objectives. Finally, if also the train station of Amsterdam Geuzenveld is included (also between Amsterdam and Haarlem), only one solution remains. The result of choosing these measures for implementation is a low value for TTT, USU and CE, but a very high value PT operating deficit. This example shows that pre-setting only 4 decision variables already may result in selecting only one Pareto solution, with consequences for objective values (in this example a very bad score for OpD). Furthermore, all other 33 decision variables are indirectly fixed to a value in this way

#### **3.4** Choosing compromise solutions

A direct method to select a preferred solution is to search for the best compromise solution, or more formally, the min-max solution (see Eq. 2, where  $\overline{Z}_w$  represents the normalized value for objective w and  $W_c$  is the compromise subset of objectives, that may also contain all objectives in W). Note that the normalization procure influences the results: choosing a suitable normalization procedure is relevant, but not considered here.

$$BCS_{W_{C}}(P) = \arg\min_{\underline{y}\in P} \left(\max_{w\in W_{C}} \overline{Z}_{w}(\underline{y})\right)$$
(2)

In Figure 8 for two different subsets of objectives the best compromise solution is plotted. First, the best compromise is sought for all four objectives (the left plot in the figure). This shows that a compromise solution exists with reasonable scores for all four objectives simultaneously: with a relative score of around 0.3, for all objectives this solution has a score in the lower end of the range. Second, the best compromise is sought for OpD and CE (the right plot in the figure). This shows that, although OpD and CE are mainly opposed, low values for both objectives are possible simultaneously. However, this comes with a price: especially TTT scores much worse when focussing only on OpD and CE.

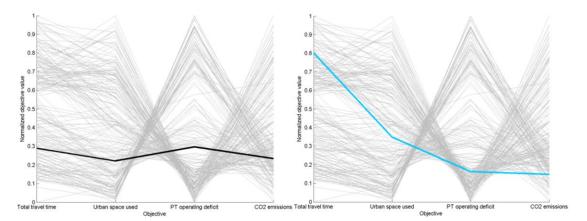


Figure 8: Compromise solution for all four objectives (left) and for OpD and CE (right)

#### **3.4.1** Defining priorities among objectives

When looking for compromise solutions, all objectives are not necessarily equally important, especially if these objectives are normalized. Instead of using equal preferences when determining a compromise solution, it is also possible to use different preferences. In a parallel coordinates plot this may be visualized as in Figure 9: all 4 objectives are 'pushed down' until only one solution remains, but instead of a horizontal line a differentiation is made per objective. In the example in the figure, the relative values of  $CO_2$  emissions and PT operating deficit are equally valued and also the relative values of total travel time and urban space used are equally valued, but the first two are seen as more important than the latter two (since the constraint for the latter two is tighter). Both relative difference in importance (Eq. 3, using coefficients for relative importance per objective  $RW_w$ ) and absolute difference (Eq. 4, using coefficients for absolute difference in importance  $AW_w$ ) may be used to choose the compromise solution. The result of applying one of these formulas is the selection of one compromise solution, representing the preferences of the decision maker concerning the four objectives considered in the analysis.

$$RBCS_{W_{c}}(P) = \arg\min_{\underline{y} \in P} \left( \max_{w \in W_{c}} RW_{w} \overline{Z}_{w}(\underline{y}) \right)$$
(3)

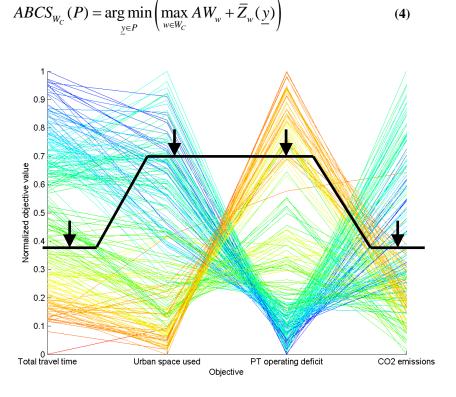


Figure 9: Constraints after optimization, differentiated per objective functions

## 4 Conclusions

In this paper an optimization framework is set up to design a multimodal transportation network, based on a predefined set of candidate locations for network developments. The framework is applied to a real world case study with 37 decision variables. The result of the optimization process is an estimation of the Pareto set, consisting of 210 possibly optimal solutions.

By this approach, a much larger area of the feasible region is investigated than would have been the case if a few pre-defined scenarios were investigated in advance, resulting in better scores on objectives. The case study shows that the framework is applicable for real life study areas and that it provides insights into trade-offs between objective values, which is useful information for decision makers.

In this specific case, a strong trade-off exists between the urban space used by parking and operating deficit: a high quality, but expensive public transport system is needed to attract former car users. Furthermore, to certain extent it is possible to reduce travel time while also reducing climate impact. This is caused by the multimodal decision variables, that promote the sustainable mode of public transport, by making it faster.

Furthermore, methods are presented to stepwise reduce the number of solutions in the Pareto set, to finally select one solution for implementation during multi-objective driven option prioritization. This can be done by setting bounds to objective function values until one solution remains or by selecting best compromise solutions (possibly by interactively selecting priorities for objectives). Another possibility is to select certain measures, that may be politically desirable, but only if these measures are included in Pareto optimal network designs, preventing inefficient decision making. Presenting the Pareto set using an interactive tool would further enable policy officers to feed decision makers with information in such way, that in one or two meetings / workshops decisions can be made. It should be noted that the operationalization of objectives (for example sustainability as  $CO_2$  emissions) is always a simplification of reality, so it is wise to define this operationalization together with policy officers. Also in that case it is still not possible to capture all desired interdependencies that exist in real world decision making, but it may be seen as a step forward to at least include the most important ones.

Further research effort may be put into analyses and methods to make more use of the Pareto set as decision support tool. This includes further identification of effective and ineffective multimodal decision variables. Another possibility is to apply pruning methods to systematically reduce the number of Pareto solutions, while remaining the main properties of the Pareto set. Finally, it would be useful to test the framework on other study areas.

## Acknowledgement(s)

The author thanks NWO for financing this research in the program Sustainable Accessibility of the Randstad. It is a part of the SRMT project. The author further thanks the interviewees who helped to develop methods for visualization that are useful in practice.

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