# A general framework for modeling intermodal transport networks

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# Abstract

Intermodal transport is receiving increased attention due to the ever-increasing demand for container transport in national and international trade, concerns on the sustainable development of the economy and the environment, and the recurring road congestion problems. In this paper, we first briefly introduce intermodal transport networks. Next, approaches for modeling intermodal transport networks are discussed. Considering different dynamic behaviors of different types of transport networks (e.g., road networks, railway networks, waterway networks, etc.), we propose a general framework for modeling intermodal transport networks. In particular, a generic intermodal transport network model is formulated for the evolution of container flows in nodes and links taking into account time dependencies. Finally, future work is discussed.

# Keywords

Intermodal transport networks; container flows; modeling

# 1 Introduction

Due to the ever-increasing demand for container transport in national and international trade, transport systems in logistic chains are facing great challenges. One crucial problem is to provide reliable and sufficient transport services in a cost-efficient way while using the current transport infrastructures. The sustainable development of the economy and the environment also require major changes and developments in transport systems, such as using geographic information systems (GIS) technology for understanding and management of freight movements, improving the market shares of railway networks and waterway networks in freight transport, etc. This requirement makes the problem even worse. One of the most promising approaches to handle this problem is to adopt the concept of intermodal transport. As a sequence, research interest in intermodal transport problems is growing steadily (Caris et al., 2008; Jarzemskiene, 2007; Macharis and Bontekoning, 2004).

Intermodal transportation may be defined as the transportation of a person or a load from its origin to its destination by a sequence of at least two transportation modes, the transfer from one mode to the next being performed at an intermodal terminal (Barnhart and Laporte, 2007). Intermodal transport networks integrate different types of transport networks available in transport systems. By integrating and coordinating the use of different transport modes, intermodal transport provides the opportunity to obtain an optimal use of the physical infrastructure so as to guarantee the operating performance of the intermodal transport network as a whole and to provide cost and energy efficient transport services.

Containers are widely used for transporting cargo in modern transport systems. For determining the optimal routing of container movement over the network, a suitable model to represent the characteristic behaviors of an intermodal transport network is necessary. In this paper, we will propose a general framework for modeling intermodal transport networks.

This paper is organized as follows. In Section 2, we first briefly introduce intermodal transport networks. Next, we analyze modeling approaches existing in the literature in Section 3. Our proposed modeling framework for intermodal transport networks is given in Section 4. Finally, Section 5 concludes the paper and presents future research directions.

# 2 Intermodal transport networks

Intermodal transport networks, as the name suggests, are the integration of different types of transport networks that are involved in the transport process of commodities and passengers. Intermodal transport is typically focused on surface transport (Arnold et al., 2004; Jarzemskiene, 2007; Southworth and Peterson, 2000). In addition, there is also some work done to make air transport an alternative option in intermodal transport (Chiambaretto and Decker, 2012; Harris et al., 2012).

An intermodal transport network can be modeled as a directed graph using two types of interconnected components, nodes and links (Rodrigue et al., 2009). Nodes represent entities like deepsea ports, inland ports, and terminals in the hinterlands. Containerized

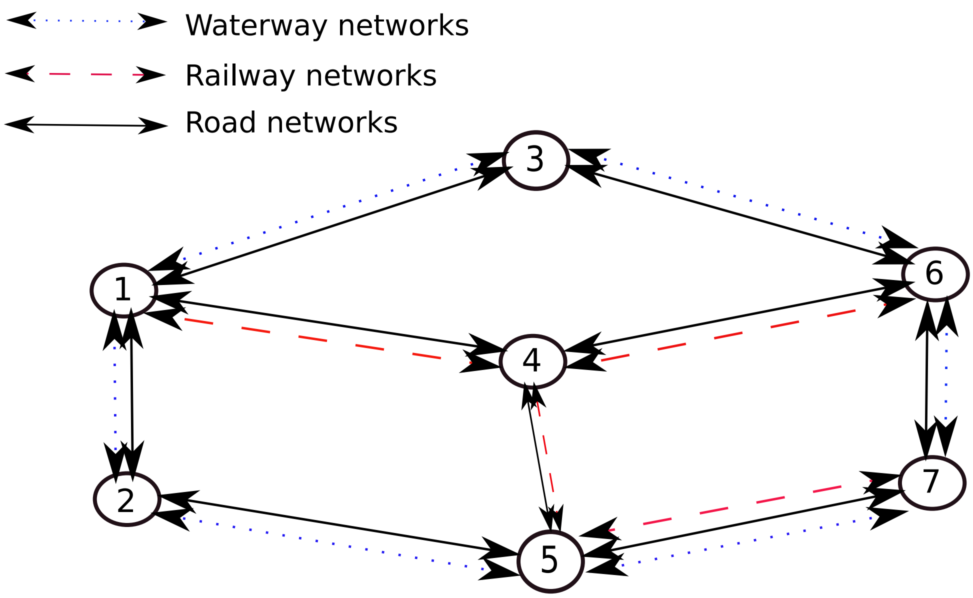


**Figure 1: An intermodal transport network. Each doubled-headed arc in the fig ure represents two directed links with opposite directions**

commodities are handled (unloaded from previous vehicles and loaded into other vehicles, or stored) at the network nodes and transported to other connected nodes (which could be their final destination or intermediate terminals on the way to their final destination) through corresponding links. Links represent entities like roads, railways, waterways, and mode transfers. Nodes are connected through links so as to form the graph that is the model of the complete intermodal transport network.

Figure 1 illustrates an intermodal transport network consisting of three individual transport networks. The road network, the railway network, and the waterway network are indicated by different colors and line styles. The structure of the graph, which is the representation of specific physical infrastructures, is determined by the connectivity of nodes within the network through links. In the operational level of planning problems in intermodal freight transport, although the freight transport capacity available in intermodal transport networks can sometimes be controlled in real-time, this particular structure is always considered to be fixed except for in emergency situations, e.g. extreme weather (Caris et al., 2008; Macharis and Bontekoning, 2004).

The different types of transport networks show some common behaviors, such as container handling operations in nodes, transport times for crossing links, etc. At the same time, they also show distinguishing behaviors because of the particular physical nature of each type of transport infrastructure and the corresponding management strategies in intermodal transport networks (Min, 1991; Southworth and Peterson, 2000). For example, the freight transport over waterway networks is slower but more environmentally friendly than that over road networks; by implementing a pre-scheduled timetable, the corresponding transport times of railway networks are much more reliable than those of road networks or waterway networks. Inspired by (Min, 1991), Table 1 gives a general comparison of different types of transport networks. When aiming for adequately describing the dynamics of intermodal transport networks, these distinguishing behaviors should be investigated.

<table>
<tr><th colspan="2"></th><th>Roads</th><th>Railways</th><th>Waterways</th></tr>
<tr><td colspan="2" rowspan="4">Similarities</td><td colspan="3">Containers are unloaded/loaded, stored, and transferred in nodes</td></tr>
<tr><td colspan="3">Transport time occurs when containers cross links</td></tr>
<tr><td colspan="3">Each link or node is subject to a certain number of constraints</td></tr>
<tr><td colspan="3">Costs occur when containers move in nodes and links</td></tr>
<tr><td rowspan="5">Differences</td><td>T.T</td><td>Short but variable</td><td>Short and fixed</td><td>Long and variable</td></tr>
<tr><td>T.C</td><td>High</td><td>Moderate</td><td>Low</td></tr>
<tr><td>E.F</td><td>Low</td><td>Moderate</td><td>High</td></tr>
<tr><td>C/A</td><td>High</td><td>Moderate</td><td>Low</td></tr>
<tr><td>Re</td><td>Moderate</td><td>High</td><td>Low</td></tr>
</table>

**Table 1: A general comparison of different types of transport networks. T.T, T.C, E.F, C/A, Re denote transport time, transport cost, environmental friendliness, connectivity/accessibility, and reliability, respectively.**

# 3 Approaches for modeling intermodal transport networks

In container transport, extra transport time and costs can occur for a pre-scheduled transport route, because containers have to wait in nodes when traffic congestion occurs. Moreover, the variation of traffic conditions in links can also cause additional transport time and costs. These situations may threaten the achievement of the stipulated delivery time for freight forwarders. On-line optimization and real-time route control allow to reduce these transport times and costs by performing real-time routing choice for the container movements at each node of the intermodal transport network with updated traffic information. However, on-line optimization and real-time route control methods require a dynamic model of the intermodal transport network.

Due to the large-scale nature of intermodal transport networks, it is impractical for the on-line optimization and real-time route control method to consider and analyze the movement of each individual container within the networks due to the high computational complexity and the high computational requirements. The similar approach as the authors did in (Baskar et al., 2009, 2010) for modeling the movement of automated vehicles in intelligent vehicle highway systems is used to consider the movement of containers in intermodal transport networks. A more aggregate model for the movement of containers should be adopted in modeling intermodal transport networks so as to obtain a trade-off between model accuracy and computational complexity of the model. Therefore, we will model the movement of containers as a flow. From the container flow perspective, the behaviors of nodes and links in transport networks are identified by the incoming container flows and the outgoing container flows. In this case, the time step size for the discrete-time model of the intermodal transport network should be chosen large enough (e.g., 1 hour) in order to capture the evolution of container flows in the network while neglecting the details, e.g., by considering the average container flows during each hour (the time step size), the discrete-time model will not need to take into account the variations of container flows at each minitue of that period of time.

An intermodal transport network model is used to deal with the problem of optimally positioning rail/road terminals for freight transport in (Arnold et al., 2004). This model gives the basic formulation of an intermodal transport network model but the time dependence of the route choice is not taken into account. In (Boardman et al., 1997), the authors consider the transport times and costs for links and mode transfers in the intermodal transport network. The models in (Arnold et al., 2004; Boardman et al., 1997), however, are static models with constant transport times and costs for links in the networks.

Some work has been done on modeling dynamics of intermodal transport networks, especially on modeling the time dependence of route choice, but the presented models have some limitations. In (Ziliaskopoulos and Wardell, 2000), the authors consider dynamic link transport times and switching delays for transport networks with multiple transport modes. The paper (Ayed et al., 2011) proposes a parallel algorithm for solving the time-dependent transport problem with a model considering time-dependent traveling times for links and time-dependent link crossing costs and transport mode transferring costs. However, the dynamic behaviors (e.g., unloading/loading containers, storing containers, etc.) of nodes in the network are not taken into account in (Ziliaskopoulos and Wardell, 2000; Ayed et al., 2011). Moreover, little work has been done about the constraints on capacities of links and nodes (e.g., the container transport capacity of links, the container storing capacity of nodes, etc.) In this paper, we investigate and model dynamics of intermodal transport networks in more detail.

# 4 A general framework for modeling intermodal transport networks

Basically, dynamics of intermodal terminals, dynamics of mode transfer connections in the intermodal terminals, and dynamics of transport connections corresponding to different types of transport networks are three important parts that need to be considered when modeling the dynamics of intermodal transport networks. In an intermodal transport network, intermodal terminals function as the container handling points where containers can be loaded, unloaded, stored, and transferred. Even through sharing some common behaviors as discussed in Section 2, dynamics of different transport connections in intermodal transport networks differ considerably because of the modes of transport being used (e.g., trucks in road networks, trains in railway networks, barges in waterway networks, etc.).

In order to develop a general framework for modeling dynamics of intermodal transport networks, it would, therefore, be reasonable to first get the generic model of intermodal transport networks with a general dynamic model for various connections. Next, the generic model can be extended to capture the individual dynamic behaviors of different transport/transfer connections based on their individual characteristics. Below, a generic intermodal transport network model will be proposed first.

## 4.1 A generic intermodal transport network model

In this paper, intermodal transport networks are considered to be the integration of road networks, railway networks, and waterway networks. Nevertheless, the generic intermodal transport network model that will be proposed here can be straightforwardly extended to include other types of transport networks, e.g., air transport networks, too.

Each single-mode transport network is represented by a directed graph $\mathcal{G}_i(\mathcal{V}_i, \mathcal{E}_i)$, $i \in \{\text{truck}, \text{train}, \text{barge}\}$ where $\mathcal{V}_i$ is a finite nonempty node set, and $\mathcal{E}_i \subseteq \mathcal{V}_i \times \mathcal{V}_i$ is the link set of all available connections among nodes within this transport network. The corresponding intermodal transport network can be represented as one directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$, where $\mathcal{V} = \mathcal{V}_{\text{truck}} \cup \mathcal{V}_{\text{train}} \cup \mathcal{V}_{\text{barge}}$ is a finite nonempty node set, $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$ is the set of transport modes and mode transfer types in the network with $\mathcal{M}_1 = \{\text{truck}, \text{train}, \text{barge}\}$ and $\mathcal{M}_2 = \{\text{truck} \to \text{train}, \text{truck} \to \text{barge}, \text{train} \to \text{truck}, \text{train} \to \text{barge}, \text{barge} \to \text{truck}, \text{barge} \to \text{train}\}$, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \times \mathcal{M}$ is the link set of all available connections among nodes. There are two kinds of links in $\mathcal{E}$, transport links and transfer links:

- A transport link $l_{i,j}^m, i \neq j$, in the link set $\mathcal{E}$ denotes that a transport connection, using transport mode $m \in \mathcal{M}_1$, from node $i$ to node $j$ exists in $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$,
- A transfer link $l_{i,j}^m, i = j, m \in \mathcal{M}_2$ in the link set $\mathcal{E}$ denotes that a mode transfer connection with transfer type $m$ exists in node $i$ of $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$.

Figure 2 shows an example of an intermodal transport network consisting of 7 nodes, 34 transport links, and 22 transfer links with transport demands $d_{1,6}, d_{4,2}, \text{and } d_{7,4}$. Each doubled-headed arc in the figure represents two directed links with opposite directions. There are three types of individual transport networks or modes of transport available in the network:

- The waterway network $\mathcal{G}_{\text{barge}}(\mathcal{V}_{\text{barge}}, \mathcal{E}_{\text{barge}})$ consists of the node set $\mathcal{V}_{\text{barge}} \in \{1, 2, 5, 6,\ 7\}$ and the link set $\mathcal{E}_{\text{barge}}$ with 8 links indicated by the dotted blue arcs in the figure.
- The road network $\mathcal{G}_{\text{truck}}(\mathcal{V}_{\text{truck}}, \mathcal{E}_{\text{truck}})$ consists of the node set $\mathcal{V}_{\text{truck}} \in \{1, 2, 3, 4, 5, 6, 7\}$ and the link set $\mathcal{E}_{\text{truck}}$ with 18 links indicated by the solid black arcs in the figure.
- The railway network $\mathcal{G}_{\text{train}}(\mathcal{V}_{\text{train}}, \mathcal{E}_{\text{train}})$ consists of the node set $\mathcal{V}_{\text{train}} \in \{1, 4, 5, 6, 7\}$ and the link set $\mathcal{E}_{\text{train}}$ with 8 links indicated by the dashed red arcs in the figure.

There are also 22 transfer links indicated by the dash-dotted green arcs in the figure connecting the waterway network, the road network, and the railway network by performing mode transfers among them.

A transport demand is defined as a group of containers sharing the origin node and the final destination node within the transport network. The evolution of each transport demand, $d_{o,d}$, over time implies the movement of a certain number of containers from their origin node $o$ to their final destination node $d$, where $(o, d)$ belongs to the set $\mathcal{O}_{\text{od}} \subseteq \mathcal{V} \times \mathcal{V}$, which is the set of all origin-destination pairs. In Figure 2, the thick yellow lines illustrate the routes of transport demands $d_{1,6}, d_{4,2}, \text{and } d_{7,4}$ in the network. For example, the route of transport demand $d_{4,2}$ is to first move from the origin node 4 to node 5 by truck, next transfer from truck to barge in node 5, then move to the

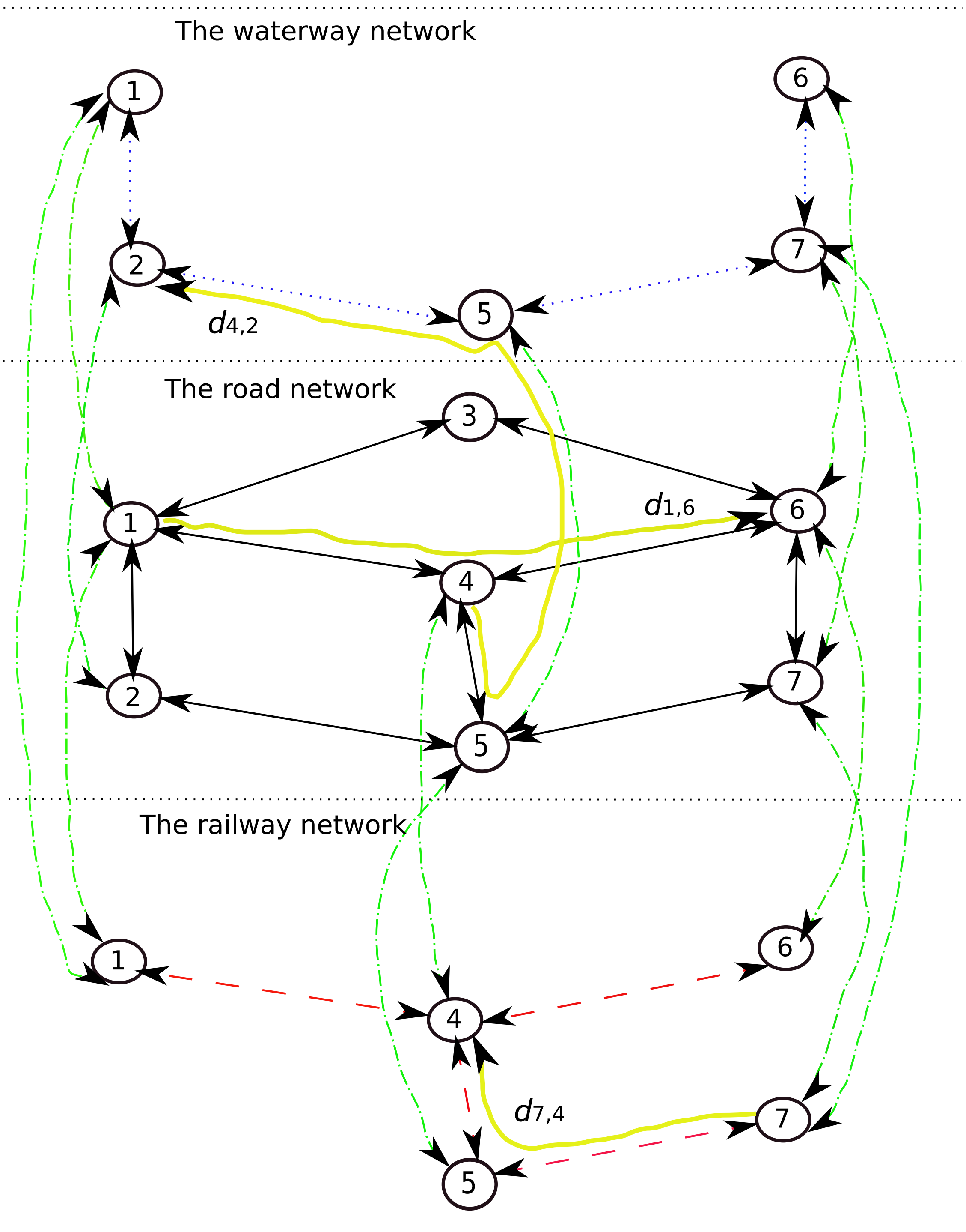


**Figure 2: An intermodal transport network with transport demands $d_{1,6}$, $d_{4,2}$, and $d_{7,4}$. The dotted blue arcs, the solid black arcs, the dashed red arcs, and the dash-dotted green arcs in the figu e indicate 8 transport links of the waterway network, 18 transport links of the road network, 8 transport links of the railway network, and 22 transfer links among three different types of transport modes (barges, trucks and trains) in nodes of the intermodal transport network, respectively. Each doubled-headed arc in the figu e represents two directed links with opposite directions**

final destination node 2 by barge; the routes of transport demands $d_{1,6}$ and $d_{7,4}$ only use trucks in the road network and trains in the railway network, respectively.

With the dynamics of an intermodal transport network, the prediction of the behavior of the network and the optimization of route choices become possible by using on-line optimization and real-time route control. Dynamics of intermodal transport networks consist of three parts - dynamics of nodes, dynamics of links, and dynamics of the interconnections among the nodes and the links within the network. Dynamics of nodes describe the evolution of the incoming and outgoing container flows associated with the nodes while dynamics of links describe those of the links. The evolution of container flows over the network is obtained by connecting container flows of both nodes and interconnected links together. These dynamics will be modeled in more detail below.

## 4.2 Nodes in the intermodal transport network

Nodes in the intermodal transport network can be categorized into three types based on the roles that they play in container transport: origin nodes, transfer nodes, and destination nodes:

- Origin nodes are the places where containers enter the intermodal transport network.
- Destination nodes are the places where containers arrive at their final destination and leave the intermodal transport network.
- Transfer nodes are intermediate terminals that containers cross on their way to reach their final destination, and in which containers could be unloaded, loaded, stored, and transferred from one mode of transport to another.

For different transport demands, a given node in the intermodal transport network might function as different types of node at the same time. For example, as shown in Figure 2, node 4 is an origin node for $d_{4,2}$, a transfer node for $d_{1,6}$, and a destination node for $d_{7,4}$ in the intermodal transport network. Therefore, the node model should be able to represent these three different behaviors at the same time.

*Node dynamics*: We consider a discrete-time model with $T_{\mathrm{s}}$ (h) as the time step size. For each transport demand $d_{o,d},\ (o,d) \in \mathcal{O}_{\mathrm{od}}$, the dynamics of transport demand $d_{o,d}$ in node $i$ can be formulated as

$$\begin{aligned} x_{i,o,d}(k+1) \quad &= \quad x_{i,o,d}(k) + \sum_{(j,m)\in\mathcal{N}_i^{\mathrm{in}}} u^m_{j,i,o,d}(k)T_{\mathrm{s}} - \sum_{(j,m)\in\mathcal{N}_i^{\mathrm{out}}} y^m_{i,j,o,d}(k)T_{\mathrm{s}} \\ &\quad + d^{\mathrm{in}}_{i,o,d}(k)T_{\mathrm{s}} - d^{\mathrm{out}}_{i,o,d}(k)T_{\mathrm{s}}, \end{aligned} \tag{1}$$

$$\begin{aligned} & x_{i,o,d}(k), u^m_{j,i,o,d}(k), y^m_{i,j,o,d}(k), d^{\mathrm{in}}_{i,o,d}(k), d^{\mathrm{out}}_{i,o,d}(k) \in \mathbb{R}_{\geq 0} \\ & \qquad \forall (o,d) \in \mathcal{O}_{\mathrm{od}},\ \forall i,j \in \mathcal{V},\ \forall m \in \mathcal{M},\ \forall k, \end{aligned} \tag{2}$$

where

- The value of $x_{i,o,d}(k)$ is the number of containers corresponding to transport demand $d_{o,d}$ and staying at node $i$ at time step $k$.

- The value of $u^m_{j,i,o,d}(k)$ $(\text{containers/h})$ is the container flow corresponding to transport demand $d_{o,d}$ and entering node $i$ through link $l^m_{j,i}$, $(j,m) \in \mathcal{N}^{\text{in}}_i$ at time step $k$. The set $\mathcal{N}^{\text{in}}_i$ is defined as

$$\mathcal{N}^{\text{in}}_i = \{(j,m) \mid l^m_{j,i} \text{ is an incoming link for node } i \text{ in } \mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{M})\}.$$

  The value of $u^m_{j,i,o,d}(k)$ equals zero when $i = o$ (which implies that node $i$ is actually the origin node $o$ of the transport demand $d_{o,d}$).

- The value of $y^m_{i,j,o,d}(k)$ $(\text{containers/h})$ is the container flow corresponding to transport demand $d_{o,d}$ and leaving node $i$ through link $l^m_{i,j}$, $(j,m) \in \mathcal{N}^{\text{out}}_i$ at time step $k$. The set $\mathcal{N}^{\text{out}}_i$ is defined as

$$\mathcal{N}^{\text{out}}_i = \{(j,m) \mid l^m_{i,j} \text{ is an outgoing link for node } i \text{ in } \mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{M})\}.$$

  The value of $y^m_{i,j,o,d}(k)$ equals zero when $i = d$ (which implies that node $i$ is actually the final destination node $d$ of the transport demand $d_{o,d}$).

- The value of $d^{\text{in}}_{i,o,d}(k)$ $(\text{containers/h})$ is the container flow corresponding to transport demand $d_{o,d}$ and entering node $i$ from the outside of the network at time step $k$. The value of $d^{\text{in}}_{i,o,d}(k)$ equals $d_{o,d}(k)$ when $i = o$, and otherwise it is zero.

- The value of $d^{\text{out}}_{i,o,d}(k)$ $(\text{containers/h})$ is the container flow corresponding to transport demand $d_{o,d}$ and arriving at the final destination node $i$ at time step $k$. The value of $d^{\text{out}}_{i,o,d}(k)$ equals $\sum_{(j,m)\in\mathcal{N}^{\text{in}}_i} u^m_{j,i,o,d}(k)$ when $i = d$ (here, we assume that containers coming from each transport demand will immediately leave the network once they arrive the corresponding destination), and otherwise it is zero.

- All variables are nonnegative arising from the nature of the transport demand $d_{o,d}$ in $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{M})$.

*Node properties and constraints*: Each node has several properties that arise from the physical infrastructure. These properties can be modeled as parameters and constraints associated with the node in the intermodal transport network model. Properties of the node include:

- The handling capacity of the equipment to unload, load, and transfer containers in the node.

- The storage capacity to store containers in the node.

The corresponding constraints in node $i$ can be formulated as:

$$\sum_{(o,d)\in\mathcal{O}_{\text{od}}} \sum_{(j,m)\in\mathcal{N}^{\text{in}}_i} u^m_{j,i,o,d}(k) \leq h^{\text{in}}_i \tag{3}$$

$$\sum_{(o,d)\in\mathcal{O}_{\text{od}}} x_{i,o,d}(k) \leq S_i \tag{4}$$

$$\sum_{(o,d)\in\mathcal{O}_{\text{od}}} \sum_{(j,m)\in\mathcal{N}^{\text{out}}_i} y^m_{i,j,o,d}(k) \leq h^{\text{out}}_i, \tag{5}$$

where

- (3) stems from the handling capacity of node $i$. The maximum handling rate of node $i$ for receiving containers from the corresponding interconnected incoming links is $h_i^{\text{in}}$ $(\text{containers/h})$.
- (4) is due to the storing capacity of node $i$. The maximum amount of containers that can be stored in node $i$ is $S_i$ $(\text{containers})$.
- (5) arises from the ability of node $i$ to connect and access links in the transport network. The maximum handling rate of node $i$ for sending containers to the connected outgoing links is $h_i^{\text{out}}$ $(\text{containers/h})$.

There are also costs associated with the container handling operations (e.g., unloading, loading, and storing) in nodes that are not included in the proposed model. We will take into account these costs in future work.

## 4.3 Links in the intermodal transport network

Each link connects nodes in the intermodal transport network and provides transport services (transporting containers between two nodes using the same mode of transport or transferring containers between two modes of transport in one node). It takes a certain period of time, called transport time, for containers to cross the link. For a given link in the network, the transport time might be fixed or vary according to the different management strategies. Moreover, the current operating conditions of physical infrastructures, such as the level of traffic density in the link, also influence the transport time.

One of the basic requirements for the container transport is to deliver containers to their destination at the stipulated time. Therefore, transport time is one crucial element that should be taken into account when analyzing the behavior of a link and also transport demands evolving over transport networks. In our model, the transport time for a given link is determined when a container enters that link and it is assumed to be fixed for this container for the remaining time that is used to get to the end of the link.

When a container enters link $l_{i,j}^m$ at time step $k$, a certain period of transport time $T_{i,j}^m(k)$ is taken to cross the link:

$$\begin{aligned} T_{i,j}^m(k) &= t_{i,j}^m(k) T_\text{s} \\ t_{i,j}^m(k) &\in \mathbb{N}\backslash\{0\} \\ t_{i,j}^m(k) &\leq t_{i,j}^{m,\max}, \end{aligned}$$

where the maximum transport time of link $l_{i,j}^m$ is $t_{i,j}^{m,\max} T_\text{s}$.

*Link dynamics:* The dynamics of each transport demand $d_{o,d},\ (o,d) \in \mathcal{O}_\text{od}$ in link $l_{i,j}^m$ can now be formulated as

$$q_{i,j,o,d}^{m,\text{out}}(k) = \sum_{k_\text{e}=k-t_{i,j}^{m,\max},\ k_\text{e}+t_{ij}^m(k_\text{e})=k}^{k-1} q_{i,j,o,d}^{m,\text{in}}(k_\text{e}) \tag{6}$$

$$x_{i,j,o,d}^m(k+1) = x_{i,j,o,d}^m(k) + \left( q_{i,j,o,d}^{m,\text{in}}(k) - q_{i,j,o,d}^{m,\text{out}}(k) \right) T_\text{s}, \tag{7}$$

where

- The value of $q^{m,\text{out}}_{i,j,o,d}(k)$ $(\text{containers/h})$ is the container flow corresponding to transport demand $d_{o,d}$ and leaving link $l^m_{i,j}$ at time step $k$.

- The value of $q^{m,\text{in}}_{i,j,o,d}(k)$ $(\text{containers/h})$ is the container flow corresponding to transport demand $d_{o,d}$ and entering link $l^m_{i,j}$ at time step $k$.

- The value of $x^m_{i,j,o,d}(k)$ $(\text{containers})$ is the number of containers corresponding to transport demand $d_{o,d}$ and presenting in link $l^m_{i,j}$ at time step $k$.

*Link properties and constraints*: There are some properties associated with each link in an intermodal transport network. These properties are:

- Transport capacity, the maximum number of containers that can stay within a link;

- Entering capacity, the maximum container flow that can enter a link.

The corresponding constraints on each link $l^m_{i,j}$ of the network $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{M})$ can be formulated as:

$$\sum_{(o,d)\in\mathcal{O}_{\text{od}}} x^m_{i,j,o,d}(k) \leq C^m_{i,j} \tag{8}$$

$$\sum_{(o,d)\in\mathcal{O}_{\text{od}}} q^{m,\text{in}}_{i,j,o,d}(k) \leq C^{m,\text{in}}_{i,j}, \tag{9}$$

where

- (8) describes that the number of containers staying inside link $l^m_{i,j}$ at time step $k$ cannot exceed the corresponding maximum $C^m_{i,j}$ $(\text{containers})$, which represents the transport capacity of link $l^m_{i,j}$.

- (9) describes that the total container flow entering link $l^m_{i,j}$ cannot exceed the corresponding maximum $C^{m,\text{in}}_{i,j}$ $(\text{containers/h})$ at time step $k$.

In general, the transport time of a given link is influenced not only by the traffic flows corresponding to the container transport demands of the network but also by the external traffic flows in that link (e.g., the traffic flows corresponding to private cars, buses, and other trucks in a link of road networks). The relationship of these two kinds of traffic flows on influencing the link transport time varies for different types of transport networks. Therefore, for links using different modes of transport, the corresponding transport times will be different. These transport times can be derived based on characteristics of each individual mode of transport. This is a topic for future work.

There are also costs associated with the container transport or the mode transfers in links that are not included in the proposed model. We will take into account these costs in future work.

## 4.4 Dynamics of the complete intermodal transport network

The evolutions of container flows on the incoming and outgoing links of the nodes in $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{M})$ are connected by

$$q^{m,\mathrm{in}}_{i,j,o,d}(k) = y^{m}_{i,j,o,d}(k) \quad \forall\, i\in\mathcal{V}, \forall (j,m)\in\mathcal{N}^{\mathrm{out}}_{i}, \forall (o,d)\in\mathcal{O}_{\mathrm{od}}, \forall k \tag{10}$$

$$u^{m}_{i,j,o,d}(k) = q^{m,\mathrm{out}}_{i,j,o,d}(k) \quad \forall\, i\in\mathcal{V}, \forall (j,m)\in\mathcal{N}^{\mathrm{in}}_{i}, \forall (o,d)\in\mathcal{O}_{\mathrm{od}}, \forall k, \tag{11}$$

where

- (10) connects node $i$ to each outgoing link $l^{m}_{i,j}$ by requiring that the value of the container flow going out node $i$ through link $l^{m}_{i,j}$ is equal to the value of the container flow entering the link $l^{m}_{i,j}$ at time step $k$.

- (11) connects each incoming link $l^{m}_{i,j}$ to node $j$ by requiring that the value of the container flow leaving link $l^{m}_{i,j}$ and entering into the node $j$ is equal to the value of the container flow entering the node $j$ at time step $k$ through link $l^{m}_{i,j}$.

Dynamics of the complete intermodal transport network $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{M})$ are driven by the transport demands $\mathcal{O}_{\mathrm{od}}$ and the routing choices for each transport demand $d_{o,d}$ at each node of the network. In our model, these routing choices are determined by minimizing the total transport time of the transport demands in the network. Therefore, the objective of this routing choice problem is defined as below (12), in which the term $J_1$ is the total transport time of the transport demands $\mathcal{O}_{\mathrm{od}}$ and the term $J_2$ is the penalty on the unfinished transport demands at the stipulated delivery time:

$$J = J_1 + J_2 \tag{12}$$

with

$$J_1 = \sum_{(o,d)\in\mathcal{O}_{\mathrm{od}}} w_{o,d} \left[ \sum_{k=1}^{N-1} \left[ \sum_{i\in\mathcal{V}} x_{i,o,d}(k)T_{\mathrm{s}} + \sum_{(i,j,m)\in\mathcal{E}} x^{m}_{i,j,o,d}(k)T_{\mathrm{s}} \right]\right] \tag{13}$$

$$J_2 = \sum_{(o,d)\in\mathcal{O}_{\mathrm{od}}} \left[ \sum_{i\in\mathcal{V}} x_{i,o,d}(N) r_{i,d} + \sum_{(i,j,m)\in\mathcal{E}} x^{m}_{i,j,o,d}(N) r^{m,d}_{i,j} \right], \tag{14}$$

where

- The value of $w_{o,d}\in(0,1]$ indicates the relative priority of the transport demand $d_{o,d}$. The relation $\sum_{(o,d)\in\mathcal{O}_{\mathrm{od}}} w_{o,d} = 1$ always holds.

- The average transport time for a container from node $i$ to destination node $d$ is $r_{i,d}$.

- The average transport time for a container from link $l^{m}_{i,j}$ to destination node $d$ is $r^{m,d}_{i,j}$.

- The planning horizon $N\in\mathbb{N}$ is a multiple of $T_{\mathrm{s}}$.

*Network dynamics:* Dynamics of the complete intermodal transport network $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$ can be formulated as an optimization problem by denoting:

$$\min_{\tilde{x}_1, \tilde{x}_2, \tilde{y}, \tilde{u}} J(\tilde{x}_1, \tilde{x}_2, \tilde{y}, \tilde{u}) \tag{15}$$

$$\text{subject to} \qquad (1) - (11).$$

where

- The symbol $\tilde{x}_1$ denotes all $x_{i,o,d}(k), i \in \mathcal{V}, (o,d) \in \mathcal{O}_{\text{od}}, k = 1 \cdots N$.
- The symbol $\tilde{x}_2$ denotes all $x^m_{i,j,o,d}(k), (i,j,m) \in \mathcal{E}, (o,d) \in \mathcal{O}_{\text{od}}, k = 1 \cdots N$.
- The symbol $\tilde{y}$ denotes all $y^m_{i,j,o,d}(k), i \in \mathcal{V}, (j,m) \in \mathcal{N}^{\text{out}}_i, (o,d) \in \mathcal{O}_{\text{od}}, k = 1 \cdots N$.
- The symbol $\tilde{u}$ denotes all $u^m_{j,i,o,d}(k), i \in \mathcal{V}, (j,m) \in \mathcal{N}^{\text{in}}_i, (o,d) \in \mathcal{O}_{\text{od}}, k = 1 \cdots N$.

This optimization problem (15) is a linear programming problem, which can be solved very efficiently using state-of-the-art solvers (Bazaraa et al., 2010).

# 5 Conclusions and future work

In this paper, by considering the common and distinguishing behaviors of transport connections or links in different types of transport networks, we have proposed a general framework for modeling intermodal transport networks. More specifically, a generic intermodal transport network model is formulated from the perspective of container flows so as to obtain a trade-off between model accuracy and computational complexity of the model.

In our future work, we will concentrate on investigating dynamic behaviors of links with different transport modes and taking into account the costs that occur assocated with the movment of container flows in the intermodal transport network. We will also work on the optimal route choice problem and the implementation of synchromodal transport.

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