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A METHOD TO QUANTIFY NETWORK OBSERVABILITY USING LINK AND ROUTE INFORMATION

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ABSTRACT

This paper extends a classic performance measure, the Maximum Possible Relative Error (MPRE), used to analyse the estimation reliability of traffic flows in a network on the basis of a limited number of sensors. Our improvement enables the analyst to consider optimal sensor locations not just for link flows, as the original metric does, but also for route and OD flows. This is an important improvement since in many applications even a perfect knowledge of link flows may not be sufficient, and lead to wrong estimations and management decisions (e.g., OD estimation).

KEYWORDS

Sensor Location Problem, network observability and coverage, MPRE, pivoting

INTRODUCTION

Information about traffic flows in a network is important to develop both short and long term strategies. However, the conditions for which the different information technologies are deployed in a network vary significantly and they often are not exploitable for all applications. It is therefore important to identify which traffic flow variables are not (sufficiently) captured, so that the opportune position of (extra) sensors could be identified.
Efficient positioning of sensors is a classical problem in Transportation and Operations Research, as well as many other disciplines involving networks: the Network Sensor Location Problem (NSLP). The problem is to determine the optimal number of traffic detectors and their position in order to obtain information on the network. A way of distinguishing NSLP approaches, following Gentili and Mirchandani (2010) is by considering (1) the sensor type (traffic counts, floating car data, cameras, etc.) and (2) the objective (OD flows, information on arcs and routes, in terms of flows and/or coverage, etc.). Further classification, within the first category, can be made by distinguishing location-based (Eulerian) sensors, which collect information on a specific point in space, and often aggregated in discrete time intervals, and trajectory-based (Lagrangian) sensors where information is variable in time and space. There are advantages and disadvantages of adopting one or the other type of sensors. For instance, Eulerian data gives complete traffic information but only for a specific point in time and space, and using aggregated parameters (mean flows, speeds and densities). Errors can be

introduced by the adopted interpolation method between the various information points, as well as because of aggregation and measurement errors. Lagrangian data provides information on complete paths but suffers of scalability, i.e. the representativity of the data depends on the share of recorded vehicles with respect to the total demand, and it is often biased (think of taxis or public transport often used as representative of the whole traffic patterns). For this reason a combination of Eulerian and Lagrangian traffic data sources is considered more and more the way to advance in this research direction (e.g., Herrera and Bayen, 2008, Van Lint and Hoogendoorn, 2009).

The second classification is given, as said, by the objective of the problem. An important distinction in this sense is between observability/coverage problems, and flow estimation problems. The first type is purely topological, as it requires only information on the network connectivity and the route set. The second type depends on the actual flows in the network and the solution will thus depend on the assumed (or observed) traffic patterns. In the literature, the NSLP was traditionally tied to the estimation of origin and destination flows from local counts. Within this view, sensor locations are identified in a way that the relative deviation between the estimated and true origin destination flows is minimized. A very popular measure to determine this deviation is the Maximum Possible Relative Error (Yang and Zhou, 1998). However, estimation errors due to the positioning of sensors can be observed and perceived also at the level of links and routes in a network. It is for this reason that recently link-based NSLP methods have been proposed (e.g., Chu et al., 2009, Castillo et al., 2009). The available approaches in this research direction focus on estimating flow observability. However, the currently available approaches aim at determining how a link flow may (partly) explain other link flows in the network. Furthermore, no approach provides a synthetic error measure to identify optimal sensor positioning.

The aim in this study is therefore twofold: (1) to bring the different objectives and sensor types used in sensor location problems within a unifying approach and (2) to provide a metric for estimating the flow estimation reliability at link, route and OD levels. In the next section we formulate the general conceptual framework and the main contributions towards this unifying theory. The advantages are described later in a simple toy network and the possible research directions are discussed.

METHODOLOGY

In networks we distinguish three flow categories: OD flows $\boldsymbol{h_w}$ ($\boldsymbol{w} \in \boldsymbol{W}$), route flows $\boldsymbol{f_r}$ ($\boldsymbol{r} \in \boldsymbol{R}$) and link flows $\boldsymbol{v_a}$ ($\boldsymbol{a} \in \boldsymbol{A}$). The (static) relations between these flows are:

$$\sum_{\boldsymbol{r}\in\boldsymbol{R}} \boldsymbol{\delta_{ar} f_r} = \boldsymbol{v_a}, \qquad \forall \boldsymbol{a} \in \boldsymbol{A}, \qquad \sum_{\boldsymbol{r}\in\boldsymbol{R}} \boldsymbol{\rho_{wr} f_r} = \boldsymbol{h_w}, \qquad \forall \boldsymbol{w} \in \boldsymbol{W}$$

The matrix elements $\boldsymbol{\delta_{ar}}$ are 1 or 0, depending on whether route $\boldsymbol{r}$ contains link $\boldsymbol{a}$ or not. Analogously the relation between OD and route flows is described by the matrix $\boldsymbol{\rho}$. Since these are linear relations, it is possible to write them in a general incidence matrix as follows:

$$\begin{bmatrix} \vec{\boldsymbol{v}} \\ \vec{\boldsymbol{h}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\rho} \end{bmatrix} \vec{\boldsymbol{f}} = \boldsymbol{I}\vec{\boldsymbol{f}}$$

The extended incidence matrix completely describes the network flow variables. However, since it takes all relations link-route-OD flows into account, it also contains duplicate information which is reflected as linearly-dependent rows. By removing the dependent rows in the matrix, only independent link flows and OD flows remain in the matrix. This procedure however is not unique in the sense that a different row order gives different results. Once all remaining rows of the incidence matrix are independent, it is possible to determine which N flows are to be observed to explain other flows by applying N iterations of Castillo's pivoting procedure (Castillo, 2009). This procedure enables one to formulate link, route and OD variables as linear relationships by opportunely swapping the element of the incidence matrix.

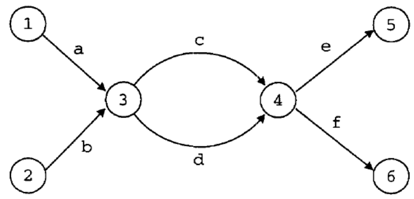

Figure 1: Toy network of 6 links, 6 nodes, 8 routes and 4 OD pairs

We explain this in a simple way by using the toy network used in Yang & Zhou (1998) and replicated in Figure 1. A solution of Castillo's pivoting process would yield, in terms of link flows, the following independent equations:

$$v_d = v_a - v_c + v_b$$

$$v_f = v_a - v_e + v_b$$

These equations show that link flows on links d and f can be fully described by the flows measured on links a, b, c and e. Similar equations can be derived using route and OD flows as well. Further details of this approach can be found in Castillo (2009). This procedure is however not unique, since a permutation of rows of the incidence matrix can result in a different observable flow selection. For instance, an equivalent solution could be found by just swapping flow variables on links c and f with d and a. It is possible however to distinguish the different solutions of the algorithm by exploiting the above flow relations. In Yang & Zhou (1998) this is done using a prior OD estimation and introducing the Maximum Possible Relative Error (MPRE). In what follows, an analogous and more general version of the MPRE for the observability problem is formulated, which does not need any prior OD.

From the pivoting procedure, a matrix $\boldsymbol{\omega}$ is obtained, which describes the relations between observed flows and unobserved flows. This matrix is obtained by removing all columns in $\boldsymbol{I}$ that do not correspond to observed flows, and taking the transpose of the resulting matrix. The unobserved true flow f and the unobserved estimated flow f^* should both satisfy the relation:

$$\sum \omega_{ij} f_j = \sum \omega_{ij} f_j^* = \tilde{f}_i, \qquad \forall i \in F$$

where $\tilde{f}$ denotes observed flows and F is the set of measured flows. This can be rewritten as

$$\sum \omega_{ij}(f_j - f_j^*) = 0, \qquad \forall i \in F.$$

Now define a function $\lambda(f_j, f_j^*) = \lambda_j$ and matrix of elements $\tilde{\omega}_{ij} = \frac{\omega_{ij}(f_j - f_j^*)}{\lambda(f_j, f_j^*)}$. Note that depending on the form of λ, $\tilde{\omega}$ will depend on the estimated flows f^*. Introducing these in the above equation gives:

$$\sum \omega_{ij} \lambda(f_j, f_j^*) = 0.$$

The norm of λ is measured by the function $G(\lambda) = \sqrt{\sum \lambda_j^2 / m}$. The MPRE is defined as the maximal norm of λ in the null space of ρ:

$$MPRE(\lambda) = \max_{\lambda} G(\lambda).$$

If λ is defined as $\lambda = (f_j - f_j^*)/f_j^*$, the general definition of the MPRE reduces to the one given by Yang & Zhou, and the elements of λ are contained in the interval $[-1, \infty[$. Analogously to the original MPRE the optimal sensor locations are found by minimizing the

MPRE. Straightforwardly, full observability is achieved when MPRE=0. An alternative definition could be introducing $\lambda = (f_j - f_j^*)/C_j$, where C_j denotes the capacity of link j. In this way, the elements of λ are bounded in the interval $[-1, 1]$. As one can see, this definition does not require any prior information about the flow in the network, but only needs a description of the network topology, or eventually the link capacities. Because of this new definition, the new MPRE becomes a powerful tool to compare different solutions of the NSLP if no a priori information is available. Furthermore, this new formulation gives the opportunity to adopt different information sources, not only local detectors, but also route and OD-based sensors. The use of the weight matrix $\boldsymbol{\omega}$ gives in addition also the possibility to give different relative importance to flow variables, or to use prior information like a target OD matrix as it was done in Yang and Zhou.

NUMERICAL EXAMPLE

Using the new MPRE definition we can distinguish the difference between placing detectors for partial observability. If in fact we have to choose three links to monitor in the toy network of Figure 1, we would be better off by choosing couples (*a*,*b,c*) rather than (*a*,*c*,*e*). The MPRE values calculated with the new method are in fact 0.882 for the first and 0.914 for the second. This is also easy to understand since on the latter choice we would not have any information on one OD pair (2->6) and one route flow (*b*->*d*->*f*) Moreover, using the above error measure, we find that, to obtain full network observability (i.e. MPRE=0), we need to monitor 4 links (*a*,*b*,*c*,*e*) and 4 routes (*a-c-e*, *a-d-e*, *a-c-f* and *b-c-e*).

CONCLUSIONS

In this paper we developed a new methodology to quantify the network flow observability based on the Maximum Possible Relative Error, used in traditional NSLP problems. We generalized this method to network state estimation, allowing one to determine which flows are or not captured in a network and to suggest where and which extra information should be placed to improve the reliability of estimation.

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