



DYNAMIC EQUILIBRIUM ASSIGNMENT CONVERGENCE BY EN-ROUTE FLOW SMOOTHING

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ABSTRACT

Upon computing the dynamic equilibrium assignment, an iterative procedure is typically required, in which route flows are averaged over successive iterations to ensure convergence. This paper shows how the rate of convergence can be improved by introducing en-route flow smoothing, allowing for en-route rerouting. At the same time, the proposed procedure solves potential problems due to grid-locks in a tractable and intuitive manner. The method is described and tested on the Sioux Falls network. In the application, when using en-route smoothing, grid-locks are resolved and a reduction is achieved of approximately 10 to 25 per cent in the number of iterations (and hence in computation time) needed to find an assignment yielding an equal duality gap. Explanations for this are given, and suggestions are made to further investigate how the procedure can be improved upon implementation.

KEYWORDS

User-equilibrium, dynamic traffic assignment, convergence, grid-lock, en-route route choice

BACKGROUND

Dynamic traffic assignment (DTA) models focus on estimating time-varying network conditions by describing route choice behavior of travelers on an infrastructure network and the way in which the traffic dynamically flows over the network. For a useful overview of DTA approaches, including a discussion on recent advances, remaining challenges, and behavioral realism in DTA research and applications, we refer to Viti and Tampere (2010). Route choice models as part of simulation-based DTA models typically describe optimizing

travel behavior (i.e., route decisions as a result of minimizing the generalized travel costs). In these models, travelers are typically assumed to choose their route from origin to destination at the time of departure and not to switch routes while travelling. This relates to the non-equilibrium pre-trip assignment since, clearly, the chosen route may not be the fastest or most attractive route when the network conditions deviate from the predicted conditions. The difference in network conditions would yield an incentive to change to a different route if the traveler was aware of the prevailing conditions. Instead of allowing en-route route changes, typically an iterative procedure is used that allows travelers to choose a different route in the next iteration, based on actually experienced route travel times and costs. Repeating this process leads to a user-equilibrium assignment in which no traveler can unilaterally switch routes and be better off (following Wardrop's equilibrium law). That such an iterative convergence procedure tends to be computationally intensive, and hence time-consuming, explains the body of research on efficient convergence methods.

Apart from the need for fast convergence, DTA models within an iterative equilibrium framework are prone to common problems with grid-locks. Typically, DTA models merely propagate travelers over the given routes and they cannot deviate from this route during the dynamic network loading process. In models using queuing and spillback, this may lead to grid-locks. Such grid-locks cause significant problems in the model, since the propagation process halts and travel times cannot be computed and no equilibrium can be determined. This problem arises mainly with intermediate route flows that have not converged yet to a user-equilibrium state and therefore some routes in a specific iteration have a too high flow rate. In practice, grid-locks may occur but are generally resolved by travelers turning around or taking detours. This is typical en-route route choice behavior, which is not modeled in DTA models that usually have only pre-trip route choice.

Past research on convergence procedures for the dynamic equilibrium assignment tend to focus on obtaining the dynamic user-equilibrium assignment as quickly as possible, i.e. to reduce the computation time (for an overview, see Mahut et al. (2008) and Mounce and Carey (2010)). In this contribution, we propose a new method to compute the dynamic equilibrium assignment which aims at both speeding up the rate of convergence and resolving the problem of grid-locks. The proposed method makes use of en-route flow smoothing to spread the route flows during the execution of the dynamic network loading model. This en-route route flow smoothing procedure is explained in the next section. Characteristics of the procedure are investigated on the Sioux Falls network in the application section thereafter. The convergence efficiency (measured here by the relative duality gap over successive iterations) of the en-route smoothing procedure is compared to that of classical convergence approaches. In the final section, we draw conclusions on the proposed convergence procedure and the presented results from the application, as well as make recommendations on how the method may be further improved upon implementation.

EN-ROUTE FLOW SMOOTHING PROCEDURE

The proposed en-route flow smoothing method is based on the hybrid route choice model introduced by Pel et al. (2009) which combines travellers' pre-trip and en-route route decisions. In short, this hybrid route choice model allows for en-route route decisions in the sense that every intersection provides travellers the possibility to decide (with some penalty) to deviate from their pre-trip chosen route when route costs on an alternative route are smaller. The ensuing of this section describes the model formulation, and how it fits into the dynamic equilibrium assignment framework.

Consider a road network $G = (N, A)$, where N is the set of network nodes (or vertices) and A is the set of network links (or arcs). A set of origin nodes $R \subset N$ and destination nodes $S \subset N$ are given. Furthermore, let the modeling time horizon be given by time interval T . Travel demand rates for a given time period $K \subset T$ is given for each origin-destination (OD) pair $(r, s) \in RS$ in terms of vehicles per hour and is denoted for each departure time instant k by $d^{rs}(k)$. The origin and destination nodes are assumed to be connected with so-called connector links to the network, which are included in set A .

First, suppose that within a certain iteration i (in the iterative equilibrium framework) all travellers have been prescribed (pre-trip) a certain route p . Let $P^{rs}(k)$ be the set of routes that are relevant for OD pair (r, s) at departure time k . The total travel demand for OD pair (r, s) which is given by $D^{rs}(k)$ is distributed according to prescribed route rates $\chi_p^{rs, (i)}(k)$ over the routes $p \in P^{rs}(k)$ where evidently $\sum_{p \in P^{rs}(k)} \chi_p^{rs, (i)}(k) = 1$. The route flows $f_p^{rs, (i)}(k)$ are then computed as

$$f_p^{rs, (i)}(k) = \chi_p^{rs, (i)}(k) d^{rs}(k). \quad (1)$$

These route flows are model input for the underlying dynamic network loading (DNL) model which simulates the traffic flows over the network and yields actual experienced (route) travel times.

We wish to emphasize that the en-route flow smoothing procedure described in the ensuing can be used in combination with essentially any iterative averaging method, such as the classical method of successive averages (MSA; first introduced by Robbins and Monroe (1951)), the route-swapping algorithm proposed by Smith and Westin (1995), and similar more recent methods like the gap function-based method by Lu et al. (2009), and gradient-based algorithm by Mahut et al. (2008) (for a comparison between MSA and route-swapping algorithms, see Tong and Wong (2010)). In this work, we use the method of successive averages. Although it is known that MSA tends to converge slowly (in particular over later successive iterations), within this study, it allows a clear and straightforward analysis of the relative benefit in convergence rate when appending the iterative averaging method with the

proposed en-route flow smoothing procedure¹. Also, it can be mentioned that the equilibrated traffic assignment as computed by MSA has desired features regarding route flow entropy maximization, or in other words, it ensures the condition of proportionality (see Bar-Gera (2010a), and Bar-Gera and Boyce (1999)).

Thus, the prescribed route rates in iteration i , used in Equation (1) to determine the route flows, are computed as

$$\chi_p^{rs,(i)}(k) = \begin{cases} \hat{\chi}_p^{rs,(i)}(k) & \text{if } i = 1 \\ \chi_p^{rs,(i-1)}(k) + \frac{1}{i}(\hat{\chi}_p^{rs,(i)}(k) - \chi_p^{rs,(i-1)}(k)) & \text{otherwise} \end{cases} \quad (2)$$

where $\chi_p^{rs,(i-1)}(k)$ are the route flow rates for the routes $p \in P^{rs}(k)$ for all OD-pairs $(r, s) \in RS$ at departure time k in the previous iteration $(i-1)$, while $\hat{\chi}_p^{rs,(i)}(k)$ are the intermediate route flow rates for the current iteration i , computed as

$$\hat{\chi}_p^{rs,(i)}(k) = \begin{cases} \frac{\exp(-\alpha^{(i)} \tilde{\tau}_p^{rs}(k))}{\sum_{p' \in P^{rs}(k)} \exp(-\alpha^{(i)} \tilde{\tau}_{p'}^{rs}(k))} & \text{if } i = 1 \\ \frac{\exp(-\alpha^{(i)} \tau_p^{rs,(i-1)}(k))}{\sum_{p' \in P^{rs}(k)} \exp(-\alpha^{(i)} \tau_{p'}^{rs,(i-1)}(k))} & \text{otherwise} \end{cases} \quad (3)$$

Here, $\tilde{\tau}_p^{rs}(k)$ is the free flow travel time on route p from r to s departing at k , and $\tau_p^{rs,(i-1)}(k)$ is the actual experienced travel time in the previous iteration $(i-1)$. Hence, the prescribed route flows are, for the first iteration, based on free flow travel times, while in next iterations these are determined by the actual experienced (route) travel times in the previous iteration. The scale parameter $\alpha^{(i)}$ in the logit model in Equation (3) determines the pre-trip smoothing in route flows. Note that lower values for $\alpha^{(i)}$ lead to a more uniformly distributed OD travel demand over the relevant routes $p \in P^{rs}(k)$. In case of computing the deterministic dynamic equilibrium assignment (as also in the application section), the scale parameter needs to be set sufficiently high.

The actual experienced route travel times used in Equation (3) can be computed as a dynamic sum of consecutive link travel times along the route,

$$\tau_p^{rs,(i)}(k) = \sum_{a \in p} \theta_a^{(i-1)}(t_a) \quad (4)$$

with $t_a = \begin{cases} k & \text{if link } a \text{ is first link of route } p \\ \theta_{a'}^{(i-1)}(t_{a'}) + t_{a'} & \text{otherwise} \end{cases}$

¹ We may remark here that preliminary results obtained from applying the en-route flow smoothing procedure in combination with a route swapping algorithm were similar to those presented in the application section (using MSA). This is in line with findings from the more elaborate comparative analysis by Tong and Wong (2010).

where $\theta_a^{(i-1)}(t_a)$ is the link travel time for vehicles entering link a at time instant t_a , in the previous iteration ($i-1$), and link a' is the previous link (i.e., prior to link a) on route p .

Now, even though these routes p have been prescribed to travellers, within our en-route smoothing procedure, they may switch routes en-route. If current traffic conditions are such that travellers are better off by deviating to another route, they might do so. In the following, travellers with the same prescribed route p can be seen as belonging to the same class of travellers. Hence, the formulation in this section is actually a multiclass formulation where each class is a distinct (pre-trip prescribed) route. Let $q \in Q^{ns}(t)$, where $Q^{ns}(t)$ denotes the set of all alternative routes q from intersection node n to the destination s at time instant t . The fraction of travellers of class p (i.e., having route p as pre-trip prescribed route) following route q is given by the probability that route q has minimal generalized route costs,

$$\hat{\chi}_{pq}^{ns,(i)}(t) = \Pr(c_{pq}^{ns,(i)}(t) \leq c_{pz}^{ns,(i)}(t), \quad \forall z \in Q^{ns}(t)). \quad (5)$$

Here $\hat{\chi}_{pq}^{ns,(i)}(t)$ is the fraction of class p travellers following route q at time instant t , based on the generalized route costs $c_{pq}^{ns,(i)}(t)$. These costs, $c_{pq}^{ns,(i)}(t)$, are the costs of following route q (which may (partially) overlap with route p) while having pre-trip prescribed route p . Here, these generalized route costs are determined by

$$c_{pq}^{ns,(i)}(t) = \theta_q^{ns,(i)}(t) + \ell_{pq} \omega^{(i)} \quad (6)$$

where $\theta_q^{ns,(i)}(t)$ is the travel time on route q from n to s , and $(\ell_{pq} \omega^{(i)})$ is the minimum improvement that is required for travellers to be rerouted. This minimum improvement depends on the cost term $\omega^{(i)}$ and the route deviation proportion ℓ_{pq} . The cost term, $\omega^{(i)}$, states that the new route q should be at least $\omega^{(i)}$ faster for travellers to be rerouted to this route. The route deviation proportion $\ell_{pq} \in [0,1]$ is the relative length of route q which does not coincide with the pre-trip prescribed route p . Consequently, we assume that the more route q deviates from the pre-trip route p , the larger the minimum improvement needs to be in order to switch routes. Note that when route q fully overlaps with (the remainder of) route p , then $\ell_{pq} = 0$, and thus $c_{pq}^{ns,(i)}(t) = \theta_q^{ns,(i)}(t)$. On the other hand, if route q deviates from route p , then $\ell_{pq} > 0$, and route q should be at least $(\ell_{pq} \omega^{(i)})$ faster in order for travellers to be rerouted to this route.

In this work, we use the instantaneous travel time to determine en-route route switching, or here called en-route flow smoothing. Since the pre-trip route fractions are based on actual experienced travel times (in the previous iteration), while the en-route flow smoothing is (currently) based on instantaneous travel times, the equilibrium state for the pre-trip assignment is not equal to the equilibrium state for the assignment with both pre-trip prescribed routes and en-route flow smoothing. However, the Wardrop user-equilibrium can still be reached by fading out the en-route flow smoothing over the subsequent iterations. Therefore, the cost term $\omega^{(i)}$ in Equation (6) is iteration-dependent. More specifically, the minimum improvement for en-route rerouting increases as the duality gap decreases, such that en-route flow smoothing becomes less likely with higher convergence.

The instantaneous route travel times $\theta_q^{ns,(i)}(t)$ can be computed as

$$\theta_q^{ns,(i)}(t) = \sum_{a \in A} \left[\delta_{aq}^{ns}(t) \left(\theta_a^{(i)}(t) + \varepsilon_a \right) \right] \quad (7)$$

where $\delta_{aq}^{ns}(t)$ is the static link-route incidence indicator (since instantaneous travel times are considered here) that equals 1 if link a belongs to route q , and zero otherwise, and the instantaneous link travel times $\theta_a^{(i)}(t)$ are computed by the DNL model. The error term $\varepsilon_a \sim N(0, \sigma_a^2)$, with $\sigma_a^2 > 0$, results in a spread of traffic flow among the (instantaneous) fastest routes.

In sum, the proposed en-route smoothing procedure allows for en-route rerouting when alternative routes provide a minimum improvement (in instantaneous route travel time). This minimally required improvement increases over successive iterations, thus fading out the effect, such that in the end the proposed procedure converges to a (pre-trip) dynamic equilibrium state similar to the dynamic equilibrium assignment computed by alternative user-equilibrium convergence procedures.

Applying the en-route flow smoothing procedure has two advantages. First of all, when grid-locks (are about to) occur, and consequently travel times increase on these road sections, travellers are able to deviate to a different route (i.e., to take a detour) thereby resolving the grid-lock conditions. The DTA model is not halted and travel times can be computed. Note that these higher travel costs on grid-lock prone road network sections will lead to different pre-trip route choice decisions, such that in the end an equilibrium still can be determined, where the equilibrium situation does not have grid-lock or circular routes. Second of all, it can be reasoned that convergence is faster by starting (within the hybrid route choice model) with en-route flow smoothing and progressively moving towards only pre-trip route decisions. Namely, procedures based on pre-trip routing converge over successive iterations, while applying the proposed en-route smoothing procedure allows the traffic assignment to converge both over successive iterations and within a single iteration (during simulation). Both these advantageous characteristics are tested and discussed in the next section.

APPLICATION

To investigate the characteristics of the proposed en-route smoothing procedure as described in the previous section, the procedure is applied to the benchmark Sioux Falls network. To this end, we applied the macroscopic whole-link dynamic network loading model including dynamic queuing and spillback, proposed and explained in Bliemer (2007). Furthermore, Equations (5)-(7) provide no closed-form expression to determine the class-specific en-route reroute fractions, which necessitates solving these by means of simulation. Here, the assumption is made of independent and identically Normal distributed link error terms leading to the Probit assignment model (see Daganzo 1979; Sheffi 1985). To limit the required number of independent consecutive draws (to replicate the link cost error distribution), low

discrepancy sequences are used. In this work, the Modified Latin Hypercube Sampling (MLHS) method is applied (for details, see Hess et al. (2005)).

Let us first briefly introduce the considered setting, after which we present the experimental setup and the numerical results.

Application description

The considered benchmark network is the Sioux Falls network, shown in Figure 1. The network layout is taken from Bar-Gera (2010b), and originally consists of 76 network links, and 24 nodes. To make the network suitable for dynamic assignment, the (original) origins and destinations are offset from the network nodes, thus creating an additional 48 connector links and 24 nodes. Additional road network characteristics (speed, capacity, number of lanes, etc.) are approximated using satellite images of the real network provided by Google Maps.

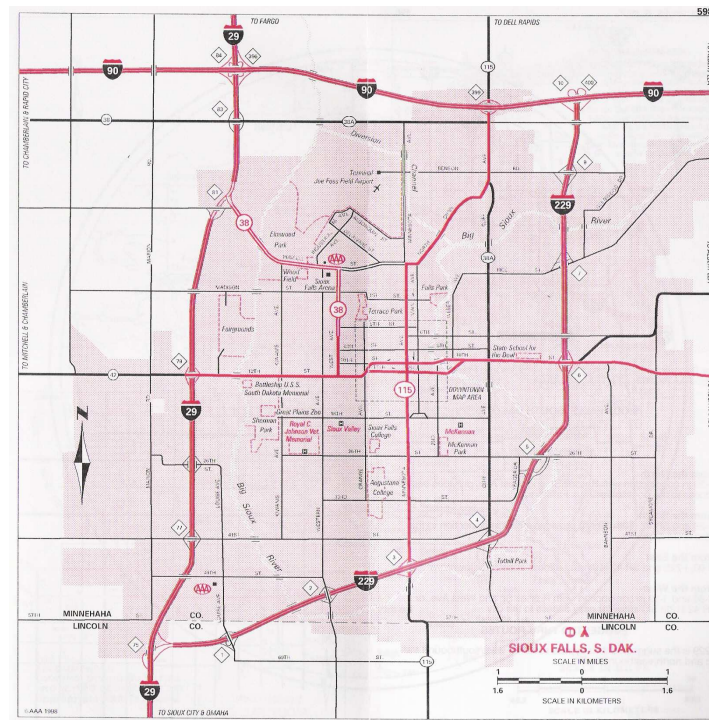


Figure 1: Sioux Falls network (source: Bar-Gera 2010b)

Table 1: Departure time profile for dynamic travel demand

From [h]	Till [h]	Fraction
00:00	00:40	0.10
00:40	01:20	0.15
01:20	02:00	0.25
02:00	02:40	0.25
02:40	03:20	0.15
03:20	04:00	0.10

A fraction (35%) of the total static travel demand (see Bar-Gera 2010b) is loaded onto the road network according to the departure time profile depicted in Table 1. The emerging road network conditions (under equilibrium assignment conditions) are shown in Figure 2.

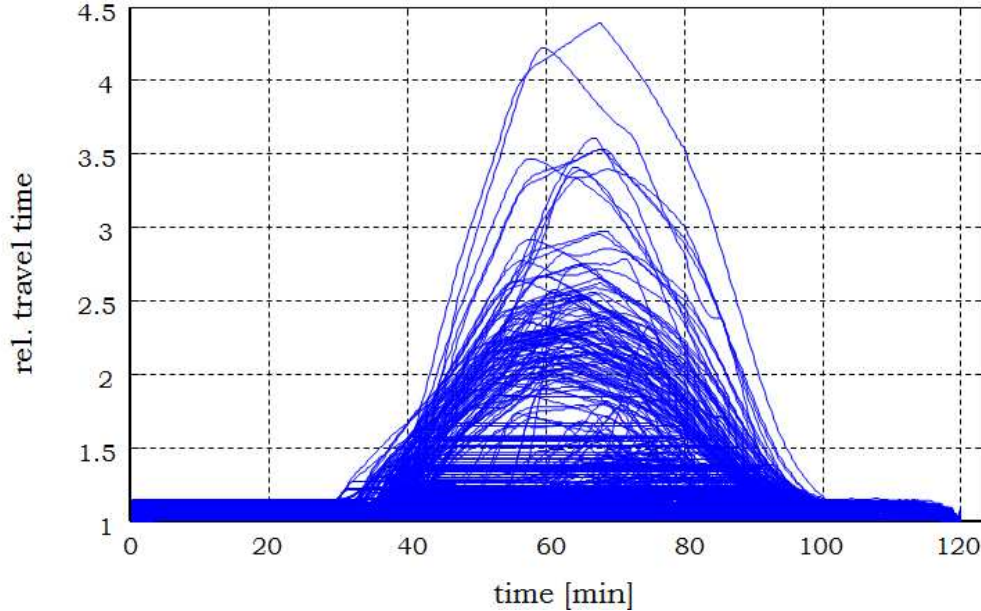


Figure 2: Network conditions (near equilibrium); route travel times relative to travel times under free flow traffic conditions as a function of departure time

Experimental setup

In the remainder of this section, we compare traditional MSA to MSA appended with the proposed en-route smoothing procedure. For both cases, the scale parameter in the logit model in Equation (3) is set sufficiently high, such that the intermediate route flow rates on the fastest route approximate 1, while these rates approximate zero on all other (slower) routes (i.e., we are considering deterministic user-equilibrium, instead of stochastic user-equilibrium). When applying en-route flow smoothing, route flow rates are updated during simulation (i.e., execution of the DNL model) based on the instantaneous travel times $\theta_q^{ns,(i)}(t)$, and a minimum improvement that is required in order to reroute $\omega^{(i)}$. The minimum improvement is set sufficiently low, such that travelers are rerouted to currently faster routes (based on instantaneous travel times), thus allowing en-route flow smoothing. To ensure reaching the equilibrium state, en-route smoothing decreases over successive iterations. To this end, the minimum required improvement $\omega^{(i)}$ increases in subsequent iterations. Here, we set $\omega^{(i)} := i \cdot 10$ minutes. Or, in words, the minimum gain for travellers to be rerouted to an alternative route is 10 minutes in the first iteration, and increases with 10 minutes in every subsequent iteration. Letting $\omega^{(i)}$ increase in this way is made based on some test runs, but was not extensively evaluated. A remark is made in the conclusions section on other ways of decreasing the en-route smoothing.

To compare the performance of these alternative convergence procedures, we use the duality gap as a measure of convergence rate. The duality gap, $\pi^{(i)}$, is given by

$$\pi^{(i)} = \frac{\int^T \sum_{(r,s) \in RS} \sum_{p \in P^{rs}(k)} [\tau_p^{rs,(i)}(k) f_p^{rs,(i)}(k)] dk - \int^T \sum_{(r,s) \in RS} [\tau_{p^*}^{rs,(i)}(k) D^{rs}(k)] dk}{\int^T \sum_{(r,s) \in RS} [\tau_{p^*}^{rs,(i)}(k) D^{rs}(k)] dk} \quad (8)$$

In words, the duality gap computes the relative difference between the total experienced travel time (by all travelers) and the system travel time that would correspond with all travelers having the travel time belonging to the shortest route for their OD-pair, denoted by $\tau_{p^*}^{rs,(i)}(k)$.

The results are presented and discussed next.

Numerical results

Before presenting the convergence results, we wish to remark here that initially some minor problems with near grid-lock conditions occurred within early iterations in case of no en-route flow smoothing. A possible explanation that in particular in early iterations (say iteration 2 to 4) traditional MSA was found to be prone to grid-lock problems is evidently that relatively *large* traffic flows are assigned to a *number* of the prevailing fastest routes. A higher number of routes being used (note that the number of used routes per OD-pair never exceeds the iteration number), basically leads to a larger probability that route flows cross in such a way that grid-lock may occur. At the same time, once grid-lock occurs, traditional MSA does not allow for rerouting. Hence, the grid-lock conditions cannot be resolved.

Grid-lock problems in the MSA settings were solved by ensuring a minimum traffic flow (even when the links downstream of the node were fully occupied). Thereby, the propagation process could continue and the DTA model was capable of computing travel times, and new (intermediate) route flow rates. This minimum traffic flow was set as $0.05/i$ of the upstream demand. Note that this ad hoc solution may ‘solve’ the problem of grid-lock, yet yields underestimated route travel times since travel times are not corrected for severe delays due to grid-lock conditions. The minimum traffic flow should not be set too high, since travel times would then be too underestimated, such that more traffic is drawn to the already congested shortest routes. On the other hand, the minimum traffic flow should not be set too low, since this will lengthen the simulation (due to lower throughput and hence higher travel times) and thus lengthen computation time. We wish to emphasize here that the proposed en-route smoothing procedure does yield correct route travel times, as the grid-lock conditions are avoided or solved in a coherent way by allowing travelers to take a detour. That this ad hoc solution in the MSA procedure leads to underestimated route travel times and hence evidently has a negative impact on the rate with which the assignment converges to the dynamic user-equilibrium is shown next.

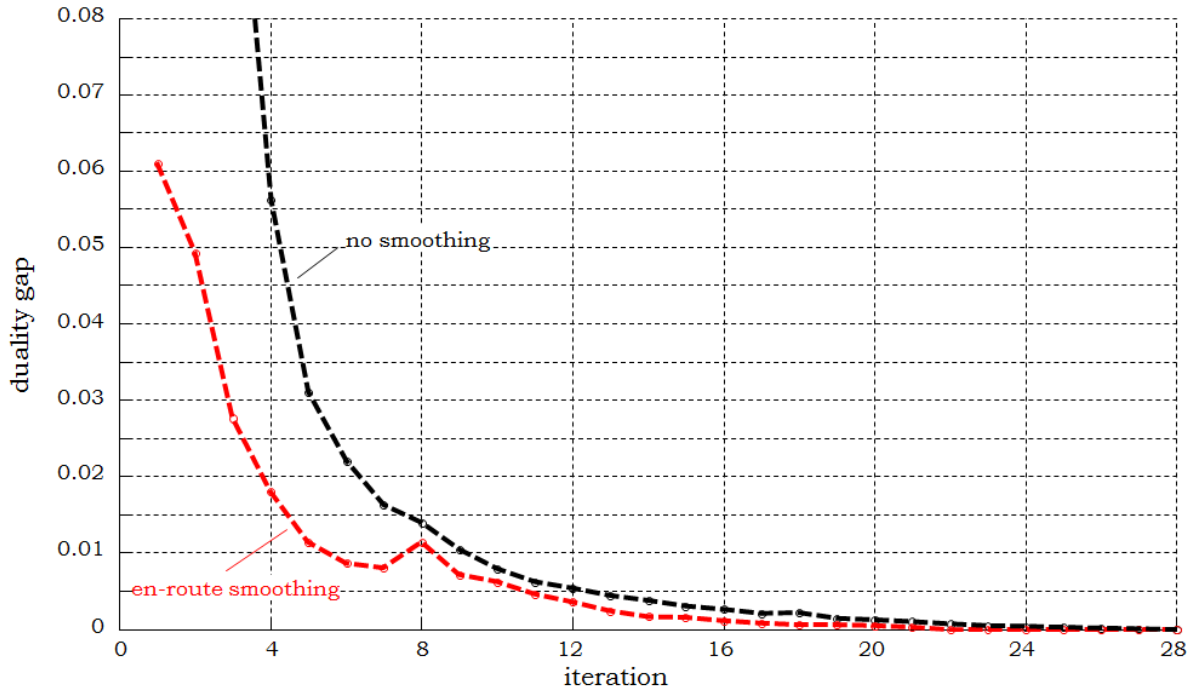


Figure 3: Evolution of duality gap by iteration; traditional MSA (black graph) and MSA with en-route flow smoothing (red graph)

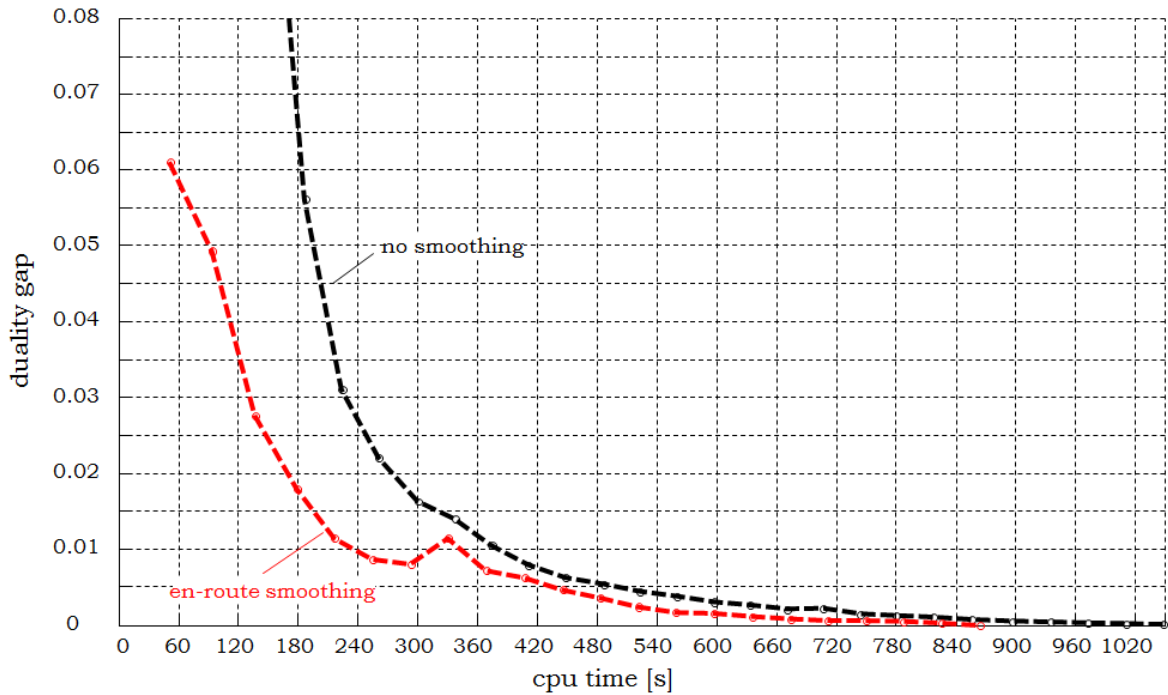


Figure 4: Evolution of duality gap by computation time; traditional MSA (black graph) and MSA with en-route flow smoothing (red graph)

The convergence speed of the various procedures, as measured by the evolution of the duality gap (given by Equation (8)) over successive iterations, is plotted in Figure 3. Note that the computed duality gap in the first few iterations in the no smoothing case (applying MSA) are

incorrect in the sense that the actual experienced travel times are underestimated due to the ad hoc solution explained in the previous paragraph to avoid grid-lock problems. From the results in Figure 3, it can be seen that applying the en-route flow smoothing procedure an assignment is found yielding a much lower duality gap. We should mention here that, in principal, the computation time of a single iteration while applying the en-route smoothing procedure increases (slightly), due to computing new route flow rates during simulation. However, two processes are at work here. On the one hand, computation time in early iterations increase somewhat as route flow rates are relatively often updated. On the other hand, the computation time is equal or slightly lower. This is due to less congested network conditions which leads to faster computation due to the way in which the DNL model is implemented (i.e., shorter simulated time horizon due to faster network clearance). Thus the overall computation time of the various convergence procedures is comparable, as a similar pattern is seen in Figure 4 showing the evolution of the duality gap as to computation time.

Evidently, the most appropriate way of investigating the beneficial effect of applying the proposed en-route flow smoothing procedure is looking at the required computation time to find an assignment with an equal duality gap. That is, to compare the two graphs in Figure 4 horizontally. This is shown in Figure 5, presenting the relative computation time of MSA appended with en-route flow smoothing as compared to traditional MSA without the en-route flow smoothing procedure. In this benchmark application, it is found that en-route flow smoothing yields a reduction of 10 to 25 per cent of computation time (for lower duality gaps, and up to 40 per cent reduction in case slightly higher duality gaps are accepted).

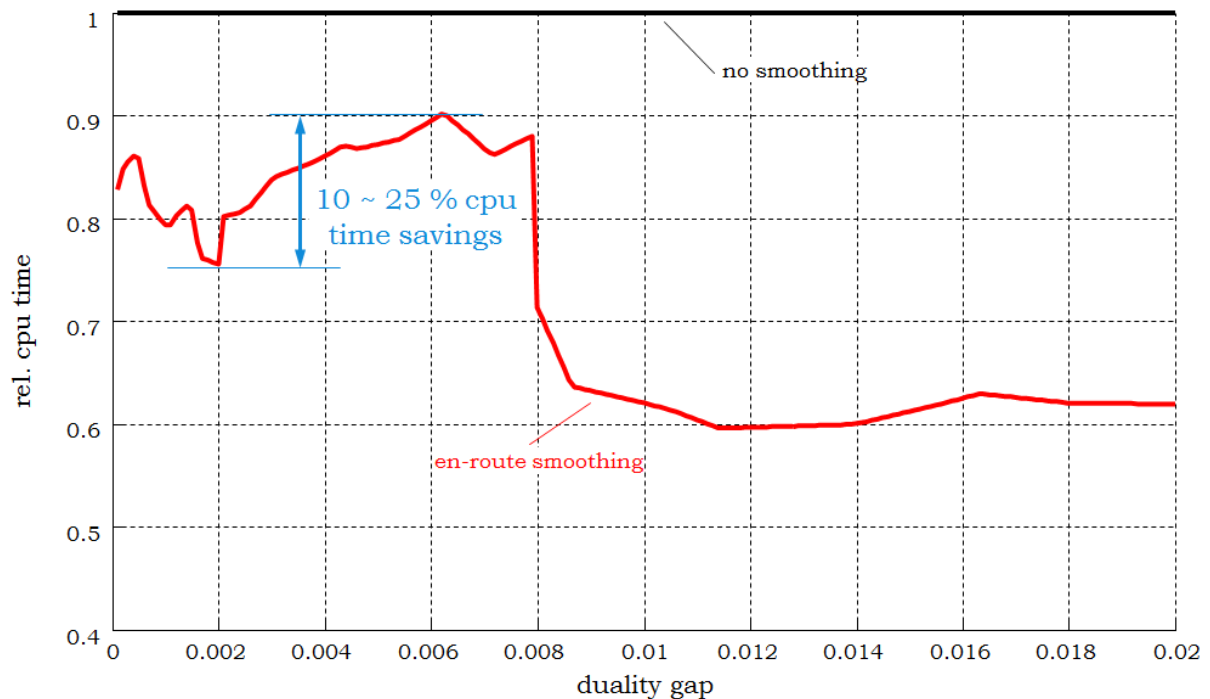


Figure 5: Relative computation time for various duality gaps; traditional MSA (black graph, set to 100%) and MSA with en-route flow smoothing (red graph)

DISCUSSION AND CONCLUSIONS

This contribution proposes a new procedure to compute the dynamic equilibrium assignment, which relies on en-route rerouting, here also called en-route flow smoothing. The procedure aims at speeding up convergence within an arbitrary iterative equilibrium framework (using, e.g., MSA), and at the same time solving common problems with grid-locks in an intuitive and coherent way. The theory and mathematical formulation of the en-route flow smoothing procedure are explained. The characteristics regarding convergence speed and grid-locks are tested on the benchmark Sioux Falls network. Based on the presented results, the following conclusions can be drawn.

First of all, the en-route smoothing procedure allows the occurrence of grid-locks to be avoided or solved. In most DTA models, travellers are merely propagated over the given routes and they cannot deviate from this route during the loading process. In models using queuing and spillback, this may lead to grid-locks. Such grid-locks cause significant problems in the model (as also in the application presented in this paper), since the propagation process halts and travel times cannot be computed and no equilibrium can be determined. Our proposed en-route flow smoothing method allows for en-route rerouting, such that when grid-lock occurs (or is about to occur), travel times on these road sections increase, and travellers are rerouted (i.e., take a detour) thereby resolving the grid-lock conditions. Higher travel costs on these road sections will lead to different pre-trip route choice decisions in subsequent iterations, such that in the end an equilibrium still can be determined (where the equilibrium situation does not have grid-lock or circular routes).

Second of all, the en-route smoothing procedure enables flow smoothing during simulation (i.e., during the execution of the DNL model) which allows for faster convergence to the dynamic equilibrium assignment. In the presented application, when applying en-route smoothing, 10 to 25 per cent less computation time is needed to find an assignment yielding an equal duality gap (for low duality gaps, and up to 40 per cent less computation time in case slightly higher duality gaps are accepted). Herein, appending the iterative averaging method (here, MSA) with the en-route smoothing procedure helps in early iterations to compute route travel times which prove to be closer to the travel times under equilibrium conditions, thereby speeding up the convergence. Since the procedure is currently based on instantaneous route travel times, the en-route smoothing is faded out over subsequent iterations to ensure that the iterative assignment correctly converges to a (pre-trip) dynamic Wardrop user-equilibrium, similar to that computed by alternative convergence procedures.

Finally, given the promising results found in this study, the mathematical formulation of the threshold for rerouting – as incorporated in Equation (6) – can be further explored.

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