The Netherlands Research School for Transport, Infrastructure and Logistics
TRAIL

Numerical diffusion in traffic flow simulations

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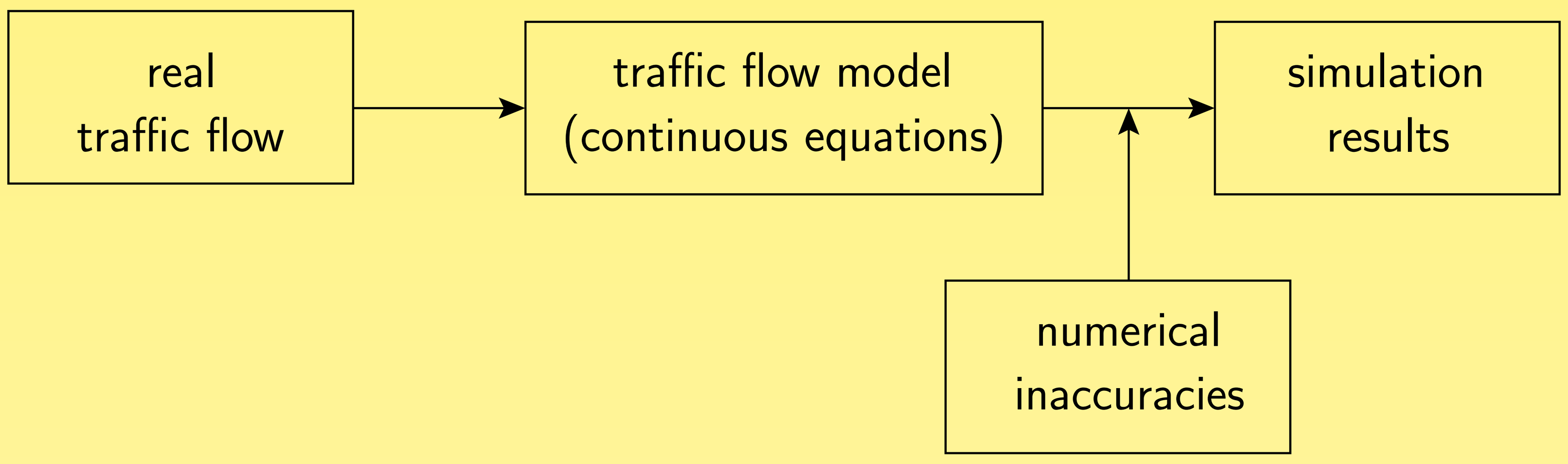
Background: traffic flow simulation



Traffic flow simulations are used in many applications for traffic flow estimation and prediction. It is well known that models only represent reality accurately to a certain extend. However, also in the step from traffic flow model to simulation results (numerical) inaccuracies occur, as is shown in the figure above. It is important to understand these inaccuracies for our understanding and interpretation of simulation results and to improve simulation methods.

The modified equation method consists of rewriting the discretized equation as a non-discretized equation. This procedure shows that the advection-diffusion equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = \underbrace{\left|\frac{dq}{d\rho}\right| \frac{\Delta x}{2}}_{\gamma} \frac{\partial^2 \rho}{\partial x^2}$$

is solved more accurately than (1). Here γ is the diffusion coefficient: if γ is large there is much diffusion, that is shocks become smooth very quickly. Applying the method to the Lagrangian formulation of (1) yields:

$$\gamma = \left|\frac{dq}{d\rho} - v(\rho)\right| \frac{\Delta x(\rho)}{2}$$

with $v(\rho)$ the vehicle/coordinate velocity (m/s).

Results

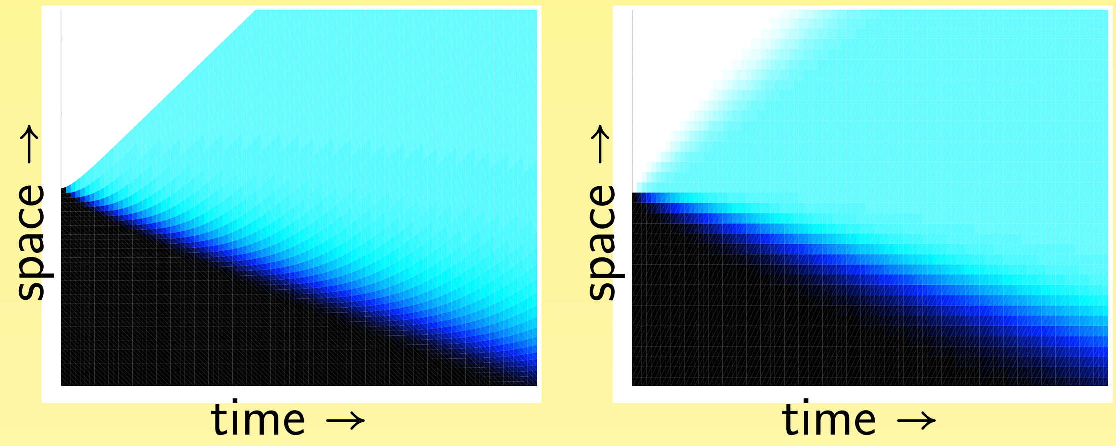
Applying the modified equation method shows that the 'amount' of diffusion depends on:

- discretization step size
- coordinate system
- current traffic state.

See also the figure on the right. Future research includes application of the modified equation method to other models, for example with other fundamental relations $q(\rho)$, and to other coordinate systems.

Numerical diffusion

Numerical diffusion is an important form of numerical inaccuracy. It causes shocks to be more smooth as shown below. It can have a great impact, for example, on whether or not congestion is predicted to spill back over a ramp.

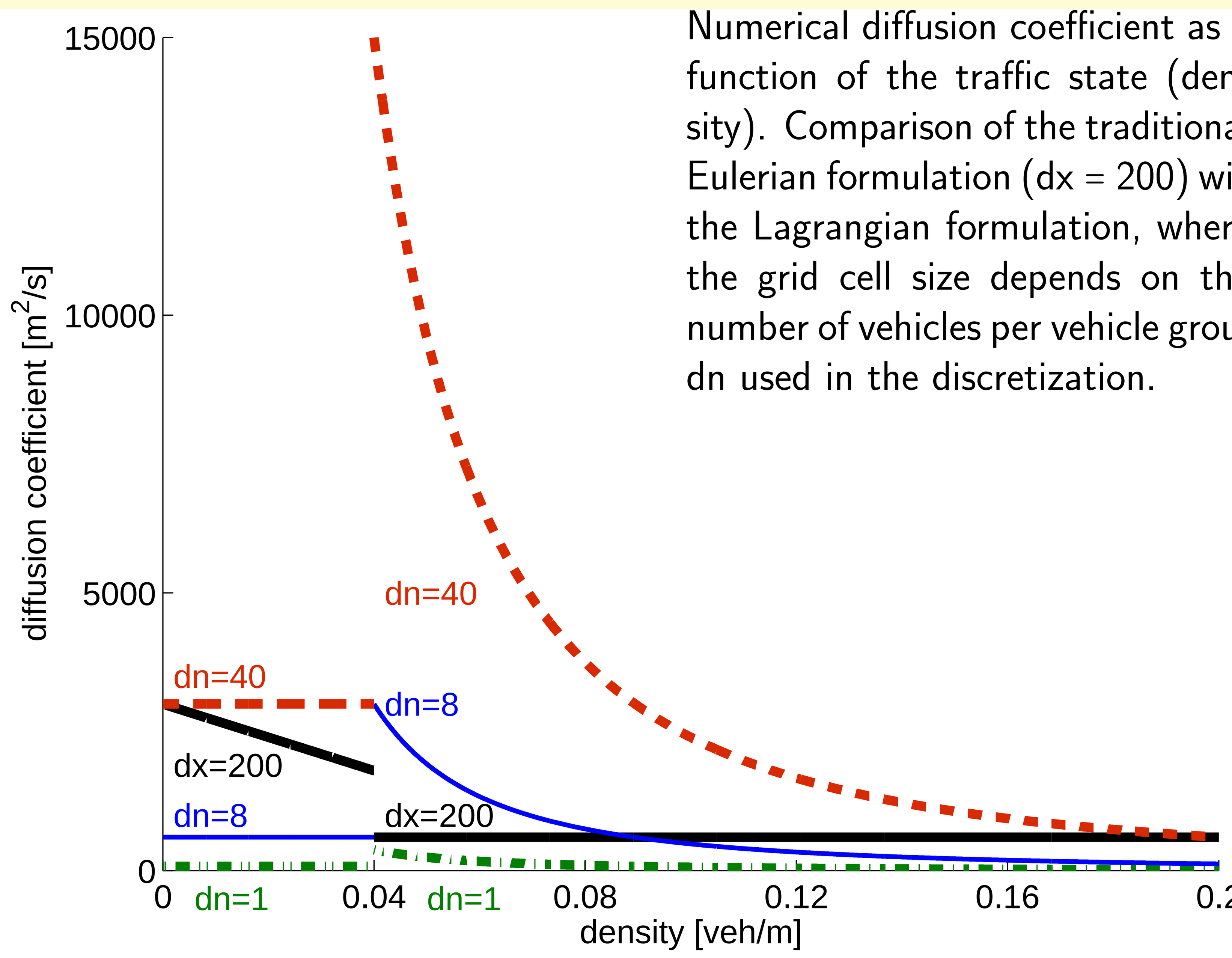


Simulation results for an initial value problem. Dark colors: high density, light colors: low density. Left: little diffusion, right: much diffusion.

The conservation of vehicles equation is used in any continuum traffic flow model:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0 \qquad (1)$$

with ρ the vehicle density (veh/m) and $q(\rho)$ the density dependent vehicle flow (veh/s). The modified equation method (see inset) is applied to analyze the amount of diffusion introduced by the widely applied discretization method based on the Godunov scheme. Moreover, the same procedure is applied to the discretized conservation equation in Lagrangian formulation, where the coordinate system moves with the vehicles.



Numerical diffusion coefficient as a function of the traffic state (density). Comparison of the traditional Eulerian formulation (dx = 200) with the Lagrangian formulation, where the grid cell size depends on the number of vehicles per vehicle group dn used in the discretization.

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