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A NEW LAGRANGIAN TRAFFIC STATE ESTIMATOR FOR FREEWAY NETWORKS

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ABSTRACT

Recent studies show that the classic kinematic wave model can be formulated and solved more efficiently and accurately in Lagrangian (vehicle number-time) coordinates than in Eulerian (space-time) coordinates. This paper investigates the opportunities of the Lagrangian form for traffic state estimation in freeway networks. We propose a new model-based state estimator where the discretized Lagrangian model is used as the model equation. This state estimator is applied to freeway traffic state estimation and validated using both synthetic data and realistic data. Different filter design specifications with respect to measurement aspects are considered. The achieved results are very promising for subsequent studies.

KEYWORDS

Freeway traffic state estimation, Lagrangian traffic flow model, extended Kalman filter.

INTRODUCTION

Freeway traffic state estimation is crucial to Dynamic Traffic Management (DTM) and real-time traffic control. Reliable and accurate state estimation promotes the performance of the whole traffic system.
Model-based state estimators are usually incorporated into traffic flow models, which are able to replicate basic traffic features. The most classic macroscopic model is the kinematic wave model (usually referred to as first-order traffic flow model or LWR model) [1] [2].The kinematic wave model is generally formulated in Eulerian (space x – time t) coordinates as a scalar conservation law [1] [2]. Its numerical solution is the so-called Godunov scheme [3], which switches between upwind and downwind schemes. Different authors have proposed traffic state estimators based on an Extended Kalman Filter (EKF), using the discretized

(Eulerian) kinematic wave model as the process model ([4], [5]). The EKF-based estimation has shown some advantages: cheap computation and adequately good results. However, the previous research ([6], [7]) has also pointed out that the EKF-based method, relying on a linearization of the state and measurement models, can cause problems. Due to the mode switching between the upwind and downwind numerical schemes [8], the Eulerian process model is highly non-linear, which makes the justification (that the process model can be locally linearized) of the EKF approach questionable, and its implementation cumbersome, particularly when traffic operates near capacity.
Recent studies show that the kinematic wave model can be formulated and solved more efficiently and accurately in Lagrangian (vehicle number n – time t) coordinates. Leclercq et al. [9] state that the main advantage of the Lagrangian approach is its exactness when the fundamental diagram is triangular. Meanwhile, traffic characteristics in this case only move in the same direction independent of prevailing traffic condition. The numerical solution then simplifies to an upwind scheme which is easy to implement and computationally less demanding. Therefore, the discretized Lagrangian kinematic wave model is less non-linear than the Eulerian model [10]. In [9] and [11], it is also shown that there is less numerical diffusion when solving the LWR model in Lagrangian coordinates.
This paper proposes a new state estimator to overcome the disadvantages of the Eulerian model while keeping the advantages of the EKF technique. This estimator is designed for freeway networks (with on-ramps and off-ramps) based on an Extended Kalman filtering framework, which uses a discretized LWR model in Lagrangian coordinates as the process model. The proposed method is tested and validated with both synthetic data and realistic data (the latter one is presented in the full paper).

METHODOLOGY

Traffic modelling of a freeway network

A first order traffic flow model in Lagrangian coordinates [11] is employed to describe the dynamic behavior of traffic flow along a freeway stretch. This model consists of two equations:

$$\frac{\partial s}{\partial t}+\frac{\partial v}{\partial n}=0 \quad \text{(Lagrangian Conservation Equation)} \qquad (1)$$

$$v=V^{*}(s) \quad \text{(Lagrangian Fundamental Relation)} \qquad (2)$$

Here, $s = 1/k$ is the average vehicle spacing (m/veh). The fundamental diagram V^* expresses speed (v) as a function of spacing (s). The vehicle number n decreases in the driving direction. Note that the variable n is not necessarily an integer.
For the convenience of state estimation, the model is discretized using an upwind scheme and an explicit time stepping scheme, as shown in (3). More specifically, traffic flow on a surveyed freeway stretch is divided into vehicle platoons of size Δn with index i ($i \in N$), while the time discretization is based on a time step t ($t \in T$) of size Δt.

$$s_{t+\Delta t}^{i}=s_{t}^{i}-\frac{\Delta t}{\Delta n}\left(V^{*}(s_{t}^{i})-V^{*}(s_{t}^{i-1})\right). \qquad (3)$$

The numerical version of the Lagrangian model on an entire freeway network then reads,

$$\begin{cases} \mathbf{s}_{t+1}=\mathbf{s}_{t}-\dfrac{\mathbf{\Delta t}}{\mathbf{\Delta n}}(\mathbf{v}_{t}-\mathbf{v}_{t}^{front}) \\ \mathbf{v}_{t}=V^{*}(\mathbf{s}_{t}), \mathbf{v}_{t}=\mathbf{q}_{t}\cdot\mathbf{s}_{t} \end{cases}. \qquad (4)$$

All boldface variables in (4) represent vectors (e.g. $\mathbf{s}_t = [..., s_t^i, ...]^T$, $i \in N$), $\mathbf{v}_t^{front}$ denotes the related successive vehicle platoon. The above equations constitute the non-linear state-space traffic flow model in Lagrangian coordinates, which can be written as:

$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{u}_t, \boldsymbol{\sigma}_t), \text{ system equation,} \quad (5)$$

$$\mathbf{y}_t = h(\mathbf{s}_t, \mathbf{r}_t), \text{ observation equation.} \quad (6)$$

Here, $f(.)$denotes the law of conservation and $h(.)$ is the fundamental relation. In (5) $\mathbf{u}_t$ depicts a vector with all inputs/disturbances (traffic demand, control signal, etc.). In (6), $\mathbf{y}_t$ depicts the new measurement with M elements. $\boldsymbol{\sigma}_t$ and $\mathbf{r}_t$ are independent zero means Gaussian random processes with covariance matrices Q_t and R_t, respectively. Here, uncertainty is introduced into the state-space model via the error covariance matrices. So any errors (mainly for zero-mean Gaussian noise) in the measurements can be incorporated into the model by properly selecting the error covariance matrix.

The essence of this model is the conservation of vehicles. In case of on-ramps (merge) or off-ramps (bifurcation) on a freeway, the influence caused by the discontinuities has to be considered. In our application, a solution for network discontinuities has been developed. The details of this method and its implementation are discussed in [12].

State estimation methods require observation models which relate observations made with traffic sensors to the system state variables. Two main (functional) data categories are identified. One is called Eulerian measurements, which collect traffic information at a fixed point, such as loop detection, video camera and radar detection. The other one is regarded as Lagrangian observations in which observers travel along with moving vehicles, such as floating car data. In Lagrangian coordinates, the Lagrangian fundamental diagram provides a natural observation model for probe vehicle data. A method to incorporate Eulerian data into the state estimator is developed and will be presented in the full paper.

Extended Kalman filter formulation and application

The new state estimator is based on the EKF technique, which uses the process model (5) in the prediction step and an observation model (6) in the correction step. Due to the advantage of the numerical scheme in Lagrangian coordinates, implementing the EKF is more straightforward than in the Eulerian case. This mainly concerns the prediction step. There is less computational demand for evolution of system states and calculation of linearization matrices at each time step. Moreover, a fully-recursive correction time scheme is used in the correction step of EKF. In this scheme, correction is performed in every intermediate time step (prediction step) within each measurement interval. This method and its advantage are presented in [13].

This state estimator is applied to freeway networks. One of the challenges in applying EKF to traffic flow models is the propagation of error over the nodes (splits and merges and/or intersections). The node models introduced in [12] are capable of doing this job, though more detailed study will be followed.

The unit of traffic in this application is a platoon with a certain amount of vehicles, as illustrated in Figure 1. The precondition is that the in-flow from both the upstream boundary and on-ramps and the out-flow to off-ramps are assumed to be known. On the basic of which new platoons are added and out-flow platoons are removed. Out-flow is always possible at exit boundary: no congestion forms at the exits.

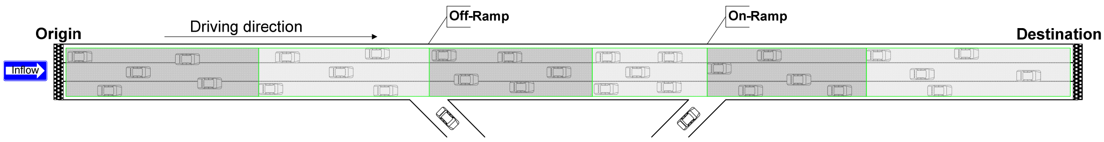
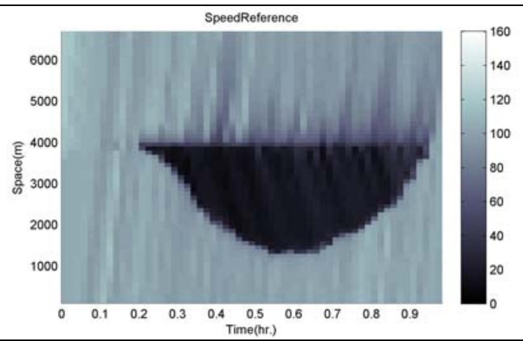
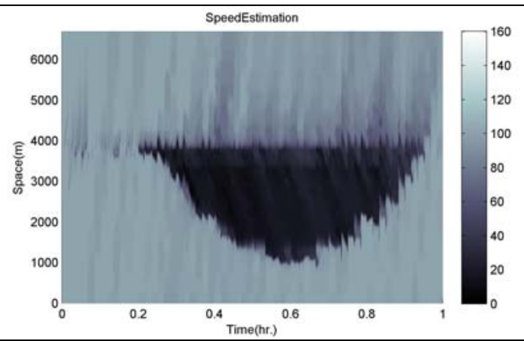


Figure 1: Discretization of traffic flow on freeway stretch

RESULTS

The first experiment with the new state estimator has been performed based on the data from a microscopic traffic simulator, FOSIM [14]. A highway stretch (similar to Figure 1) of about seven kilometers in length with a single on-ramp and an off-ramp has been simulated over 1 hour. There is a bottleneck in the middle (starts from the on-ramp). The demand during the simulation period varies, so that a jam emerges and dissolves at the bottleneck. Both the Eulerian (loop) measurements and the Lagrangian (floating car) measurements are considered as input to the observation model of EKF.
Figure 2 shows the speed maps from both the ground truth data and the state estimation based on a limited amount of loop detection (500m). The speed map (b) derived from the state estimator is comparable to the reference case (a) based on the FOSIM simulation. The starting time and dissolving time of congestion are similar to the reference plot. The typical trend of traffic propagation is also present in the estimation.





(a) Reference case	(b) State estimation

Figure 2: Speed maps from simulation

The full paper will present more detailed results as well as a simulation with realistic traffic data. Nonetheless, the results presented here already show that the new state estimator is capable of reproducing the traffic condition to a satisfactory extent. Moreover, a comparison with the Eulerian approach will be further considered.

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