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# OPTIMAL CONFIGURATION IN A PUZZLE-BASED COMPACT STORAGE SYSTEM

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## ABSTRACT

This paper studies random storage in a puzzle-based compact storage system where products are stored multi-deep. Although such storage systems are still rare, they are increasingly used, for example in automated car parking systems. Each load is accessible individually and can be moved in x- and y-directions by a shuttle as long as an open slot is available next to it, comparable to Sam Loyd's sliding puzzles. A lift moves the loads in the z-direction. We derive the expected travel time of a random load from its storage location to the input/output point. We optimize system dimensions by minimizing the retrieval time.

## KEYWORDS

Logistics, warehousing, puzzle-based compact storage system, random storage

## INTRODUCTION

Storage facilities, including warehouses, distribution centers and container terminals, can be found everywhere in supply chain networks. They form the key nodes in supply chain networks decoupling demand from supply in time and quantity. Over the past decades, these facilities have evolved towards higher storage density, more automation, and more intelligent control. This development is particularly attractive as (1) land becomes more scarce and expensive in many densely populated areas, like the Netherlands, or when space is scarce, like within ships or aircrafts; (2) costs of technology-based systems are decreasing, and (3) system response times have to be shortened to earn customers. As a result, a new generation of storage systems is emerging: dense, autonomous and intelligent (DAI) storage systems also called puzzle-based compact storage systems. In a puzzle-based compact storage system, every load is accessible and can be moved between storage locations and input/output (I/O) point. Each unit load can move in x- and y-directions as long as an empty slot is available next to it. In such a system, unit loads are stored in a grid in which at least one location is open, and so the location utilization can reach $(n-1)/n$, where $n$ is the number of storage cells in the grid (Gue and Kim, 2007). Compared to traditional storage facilities where unit loads are stored single deep with many transport aisles, puzzle-based compact storage systems need less space.

The random storage policy is studied broadly in the literature (Hausman et al., 1976, Bozer and White, 1984, Lee and Elsayed, 2005, De Koster et al., 2008 and Yu and De Koster, 2009). Random storage requires the least data since no product information is used in determining storage assignment (Goetschalckx and Ratliff, 1990).In many studies, e.g., Hausman et al. (1976), and Lee and Elsayed (2005), it is used as a benchmark to measure the improvements of other storage policies.

In this study, we will derive retrieval time models to estimate the performance of a random storage policy and use these to optimize puzzle-based system dimensions yielding minimum system response times.

# PROBLEM DESCRIPTION

The main components of a puzzle-based compact storage system are storage racks, shuttles, a lift, and an I/O-point. Shuttles can move in x- and y- directions while carrying a unit load. A shuttle can move a unit load as long as there is an empty slot next to it. Shuttles move unit loads into the open locations to maneuver the desired unit load to the output at the load's storage level. A lift takes care of movements in the z- direction. At each level, the shuttles can move independently of the lift. We assume the I/O point is located at the lower left corner of the system. When idle, the lift waits at the I/O point.

This paper optimizes dimensions of a puzzle-based compact storage system under random storage policy. In order to do this, we define a mathematical model for the expected retrieval time of a random unit load as a function of system dimension sizes. Figure 1 illustrates a schematic view of a puzzle-based compact storage system with lift.

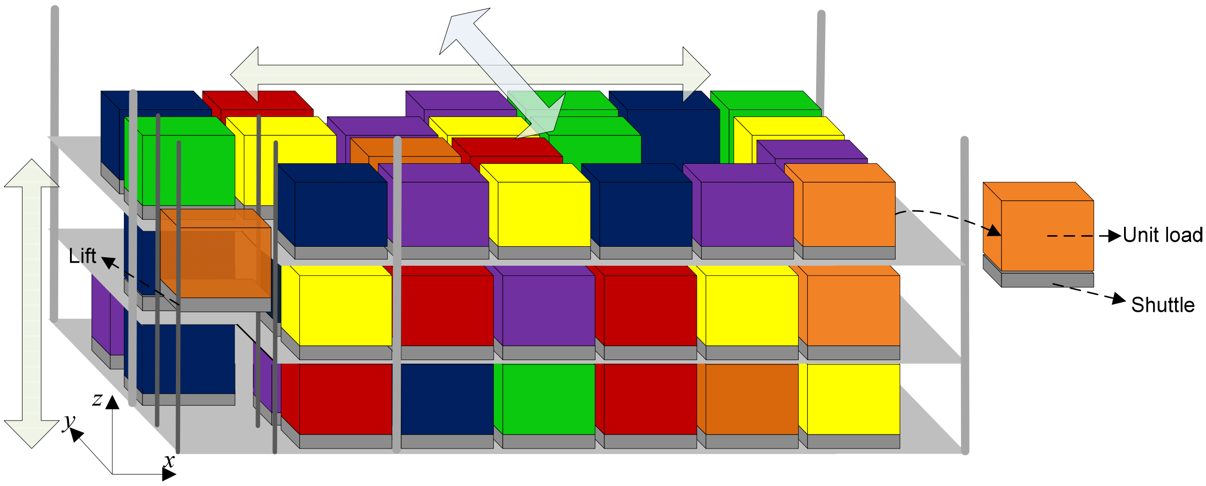


**Figure 1:** A puzzle-based compact storage system with a lift

# MATHEMATICAL MODEL

The basic assumptions we use in the model are commonly used by other papers (see, Hausman *et al.,* 1976, Yu and De Koster, 2009). A random retrieval location can be denoted by $(X, Y, Z)$ where $X$, $Y$ and $Z$ refer to coordinates in x-, y- and z- directions respectively. The system capacity is a known positive constant. A random storage policy is assumed. It is also assumed that the utilization of the system cannot exceed $(V' - \max\{L,W\})/V'$, where $V'$, $L$, and $W$ represent the capacity of the system in number of storage locations, the number of columns in each level, and the number of rows in each level, respectively.

**Theorem 1**. *If there is at least one empty location in each row and each column of each level of a puzzle-based compact storage system (i.e. max utilization* $\leq (V' - \max\{L,W\})/V'$ *), the minimum retrieval time of a random unit load stored at location* ($X$, $Y$, $Z$)*, can be estimated by the following equation:*

$$RT(X,Y,Z) = \max\{X+Y, Z\} + Z \,. \tag{1}$$

**Proof**. Theorem 1 can be proven by using mathematical induction, which is omitted here.

Using this theorem, we obtain the expected retrieval time given by Equation (2) and the mathematical model of the problem as below (Model MGM):

$$\min ERT \,, \tag{2}$$

subject to:

$$l \times w \times h = V \,, \tag{3}$$

decision variables: $l > 0, w > 0, h > 0$ ,

where $l$, $w$, and $h$ (length, width, height of the system) are the decision variables. All the dimensions are expressed in time units. $V$ represents the volume of the system in cubic time unit.
Constraint (3) makes sure the given capacity is achieved.
The model is non-linear and mixed integer; however, we can optimally solve it by splitting it into several solvable sub-models and reducing the feasible area of the decision variables without losing the optimal solution.
In order to solve the model we have to derive the expected retrieval time in Equation (2). The expected retrieval time for any puzzle-based system with a given capacity can be calculated as follows:

$$ERT = \int_{t=0}^{\max\{w+l,h\}+h} t f(t) dt \,, \tag{4}$$

where, $t$ represents the retrieval time for any retrieval location. $f(t)$ represents the probability density function of retrieval time $t$, $0 \leq t \leq \max\{w+l, h\} + h$. In order to calculate the expected retrieval time, we need to derive $f(t)$. By knowing the cumulative distribution function of the retrieval time ($F(t)$) we can then derive $f(t)$. The cumulative distribution function can be calculated as follows:

$$F(t) = P(T \leq t) = P(\max\{X+Y, Z\} + Z \leq t) = P(X+Y+Z \leq t \cap 2Z \leq t) \,. \tag{5}$$

The two conditions, $X+Y+Z \leq t$ and $2Z \leq t$ are not independent of each other and therefore cannot be separated. By simultaneously considering these two conditions, the calculation of ERT can be done into four different complementary cases, each referring to a specific configuration of the system. The four cases of system configuration are listed as follows:

- Case A: $h \leq w$,
- case B: $w < h \leq l$,
- case C: $l < h \leq l + w$,
- case D: $l + w < h$.

The classification is due to different ways of calculating the probability density function in each case other than the other cases.

# FINDINGS AND CONCLUSIONS

We obtain the optimal solution of Model MGM by comparing the solutions of four cases. The following equations give optimal values of $ERT_A$, $ERT_B$, $ERT_C$, and $ERT_D$ as a function of volume of the system *v*.

$$ERT_A^* = 1.53097\ V^{1/3} \quad (6)$$

$$ERT_B^* = 1.53789\ V^{1/3} \quad (7)$$

$$ERT_C^* = 1.54167\ V^{1/3} \quad (8)$$

$$ERT_D^* - 1.01009\ V^{1/3} \quad (9)$$

As it can be seen from Equations (6-9), the solution of case A ( $h \le w$ ) gives the minimal ERT for Model MGM. Therefore, the following equations define the optimal solutions of Model MGM. For any given volume of the system, a system with the following dimension sizes is the system with minimum expected retrieval time.

$$h^*(V) = \left(-2 + \left(\frac{1}{2}(11 + 3\sqrt{13})\right)^{1/3} + \left({}^{2}\!/_{11 + 3\sqrt{13}}\right)^{1/3}\right)^{1/3} V^{\frac{1}{3}} \approx 0.874461 V^{\frac{1}{3}} \quad (10)$$

$$w^*(V) = l^*(V) = \left(-2 + \left(\frac{1}{2}(11 + 3\sqrt{13})\right)^{1/3} + \left(\frac{2}{11 + 3\sqrt{13}}\right)^{1/3}\right)^{1/3} v^{1/3} \approx 1.069374 V^{\frac{1}{3}} \quad (11)$$

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