

Maintenance in the Rolling Stock Rescheduling Model

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Abstract

This paper addresses the Rolling Stock Rescheduling problem (RSRP), while taking maintenance appointments into account. After a disruption, the rolling stock has to be rescheduled to uphold as much of the passenger service as possible. The RSRP has been tackled in the literature, however a limited number of rolling stock units have a scheduled maintenance appointment somewhere during the day. In this paper we propose three different models to take the maintenance appointments into account in the RSRP.

The first model is the Extra Unit Type model. This is an extension of the Composition model, by adding extra unit types for every rolling stock unit that requires maintenance. In this way constraints can be set on the maintenance appointments.

The second model is the Shadow Account model, which keeps track of a shadow account for all the units that require maintenance. In this way, constraints can specifically be set on the units with a maintenance appointment.

The final model is the Job-Composition model, which is a combination of the Job model and the Composition model, both known in the literature. Paths are created such that maintenance units are on time for their appointment.

Currently, only the first two models are tested on real life instances from Netherlands Railways and results show that the Shadow Account model performs better in case there are many possible composition changes.

1 Introduction

During an extensive planning period rolling stock units are appointed to trips to supply passenger demand. A selected amount of these train units are scheduled to have a maintenance check somewhere during the day at one of the maintenance locations. These maintenance locations are positioned at a limited number of stations. During the planning phase there is enough time to adjust the rolling stock schedule in such a way that the rolling stock units that require maintenance are in time for their maintenance appointment, see for instance Maróti and Kroon [3].

During the daily operations, the railway network is exposed to disruptions. A disruption blocks part(s) of the infrastructure, causing the timetable and resource schedules (e.g. rolling stock and crew schedule) to be infeasible. Disruption management is then required to uphold as much of the passenger service as possible. In this paper the focus is on the Rolling Stock Rescheduling Problem (RSRP), which aims to find a new rolling stock schedule, that upholds as much of the passenger service as possible, and also as swiftly as possible.

In the Netherlands different rolling stock types are available to transport passengers. These rolling stock types differ in the passenger capacity and there are multiple units of every type available. Current rolling stock rescheduling models assume that every rolling stock type have

interchangeable rolling stock units. That means that there is no distinction between units that require maintenance and units that do not. As a result the train units scheduled for maintenance will most likely not be in time for their maintenance appointment after a disruption. This could cause problems, train units are no longer allowed to be employed for passenger transportation after a threshold amount of time without a maintenance check.

In this paper, first a literature overview on the RSRP is given in Section 2. Then, three different approaches for including maintenance in rolling stock rescheduling models are given. First, the Extra Unit Type model is discussed in Section 3. Secondly, the Shadow-Account model is presented in Section 4 and finally the Job-Composition model is proposed in Section 5. Then, in Section 6 two of the models are tested on real life instances of Netherlands Railways (NS) and compared with respect to the computation time.

2 Literature research

In this section we will briefly give an overview of the existing literature on the RSRP and on taking maintenance into account in rolling stock scheduling models.

Fioole et al. [1] formulated a model to assign rolling stock to the timetable. The model is able to handle complicated line structures, such as combining and splitting of trains. NS uses this model to generate the rolling stock schedules since 2004.

Maróti and Kroon [3] and Maróti and Kroon [4] proposed two integer models for maintenance routing of rolling stock: the “Transition Model” and the ”Interchange Model”. Both models first schedule the rolling stock without taking maintenance into account and thereafter exchanging unit duties such that maintenance requirements are met.

Nielsen [6] extended the model of Fioole et al. [1] to cope with rescheduling. He formulated an integer programming model with the adjusted timetable and the original rolling stock schedule as input, and an adjusted rolling stock schedule as output. This model will be used as base stock model in this paper and is referred to as Composition model.

Nielsen et al. [5] propose a rolling horizon to solve the rolling stock rescheduling problem. The idea behind the rolling horizon is that at the beginning of the disruption not all information about the duration of the disruption is known: this information becomes gradually available. The rescheduling is periodically performed within a limited rolling horizon length, possibly taking new information into account. At each time instant where an updated timetable becomes available, or when a certain amount of time has passed without any update, the MIP is solved for a rolling stock horizon time window. This model is tested on instances of NS. Solutions with small deviations from the original plan are found in a short time.

Kroon et al. [2] consider real-time rescheduling of rolling stock during large disruptions while taking dynamic passenger flows into account. They use a two-stage feedback loop, where in one stage the rolling stock allocation is optimized by using the model of Nielsen [6] and in the other stage the effect of passenger flows on the allocation of the rolling stock is determined by means of simulation. This simulation provides feedback in terms of passenger delays due to limited capacity of the assigned rolling stock. This feedback is then used in the optimization model in order to reallocate the rolling stock again, in such a way that the passenger delay is reduced. Given the reallocation of the rolling stock, the passenger simulation is performed again and feedback is given to the optimization model. This process continues for a number of iterations. This model is tested on several cases of NS. The computation times are acceptable.

3 Extra train components for maintenance

In this section the first model to take maintenance into account is presented. From now on this model is called the Extra Unit Type (EUT) model. The EUT model is based on the Composition model, so this section begins with explaining the Composition model in detail. The Composition

model does not take maintenance into account, because it assumes every rolling stock unit of the same type to be interchangeable.

3.1 Composition model

Let T be the set of trips in the timetable and S the set of stations. A trip is defined as a train driving from one station until the first station where the composition of the train can possibly be changed. Denote $s_t^{dep}(s_t^{arr})$ as the station where trip $t \in T$ starts (ends). Furthermore, let τ_t^d be the departure time of trip $t \in T$ and τ_t^a the arrival time and let $\sigma(t)$ be the successor of trip t , see for instance Figure 1 where a trip t between stations A and B is succeeded by a trip $\sigma(t)$ between the stations B and C .

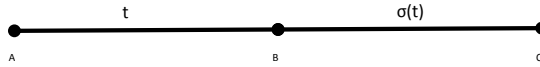


Figure 1: The successor of trip t

Let M be the set of rolling stock types and denote P as the set of possible compositions, where a composition is a combination of train units that can be used on a trip. Note that the empty composition can be assigned to a trip, meaning that the trip is cancelled. Then $v_m(p)$ denotes the number of train units of type $m \in M$ in composition $p \in P$. N_p denotes the total number of units in composition p , and R is the set of allowed composition lengths. For each trip $t \in T$, $\eta(t)$ denotes the set of allowed compositions on the trip.

At the end of a trip the composition of a train can possibly be changed, depending on the shunting rules at the station, before departing on its successive trip. A composition change denotes the composition of the incoming and outgoing trip and which units are coupled and uncoupled during the composition change. Furthermore, $\rho(t)$ denotes the allowed composition changes at the end of trip $t \in T$, p_q denotes the incoming composition of a trip when composition change q is used, and o_q denotes the outgoing composition when composition change q is used. For instance, a composition change takes place at station B in Figure 1, then p_q denotes the composition assigned to trip t and o_q denotes the composition assigned to trip $\sigma(t)$.

For a given composition change $q \in \rho(t)$, $\alpha_{q,m}$ denotes the number of uncoupled units of type $m \in M$ in this composition change and $\beta_{q,m}$ denotes the number of coupled units of type $m \in M$ in this composition change. The time at which coupling takes place at the end of trip $t \in T$ is denoted by τ_t^+ and the time at which an uncoupled unit is available after uncoupling is denoted by τ_t^- . Note that in the Netherlands it is not allowed to both couple and uncouple at the end of the same trip.

The available number of units $m \in M$ at station $s \in S$ at the beginning of the planning period is denoted by $i_{s,m}$ and the desired number of available units of type $m \in M$ at station $s \in S$ at the end of the planning period is given by the parameter $i_{s,m}^\infty$.

The model further requires the following decision variables:

- $X_{t,p} \in \{0, 1\}$ denotes whether composition $p \in \eta(t)$ is used on trip $t \in T$.
- $Z_{t,q} \in \{0, 1\}$ denotes whether composition change $q \in \rho(t)$ is used at the end of trip $t \in T$.
- $I_{t,m} \in \mathbb{Z}_+$ denotes the number of units of type $m \in M$ in the inventory at station s_t^{dep} at time τ_t^+ .
- $C_{t,m}$ and $U_{t,m} \in \mathbb{Z}$ denote the number of units $m \in M$ that are coupled and uncoupled at the end of trip $t \in T$.

- $D_{s,m}$ denotes the deviation from the desired end of day balance on station $s \in S$ for rolling stock type $m \in M$.

Model:

$$\min f(X, Z, D) \quad (3.1)$$

subject to:

$$\sum_{p \in \eta(t)} X_{t,p} = 1 \quad \forall t \in T \quad (3.2)$$

$$X_{\sigma(t),p} = \sum_{q \in \rho(t): o_q=p} Z_{t,q} \quad \forall t \in T, p \in \eta(t) \quad (3.3)$$

$$X_{t,p} = \sum_{q \in \rho(t): p_q=p} Z_{t,q} \quad \forall t \in T, p \in \eta(t) \quad (3.4)$$

$$C_{t,m} = \sum_{q \in \rho(t)} \beta_{q,m} Z_{t,q} \quad \forall t \in T, m \in M \quad (3.5)$$

$$U_{t,m} = \sum_{q \in \rho(t)} \alpha_{q,m} Z_{t,q} \quad \forall t \in T, m \in M \quad (3.6)$$

$$I_{t,m} = i_{s_t^{dep},m} - \sum_{t' \in A_t} C_{t',m} + \sum_{t' \in B_t} U_{t',m} \quad \forall t \in T, m \in M \quad (3.7)$$

$$I_{t,m} \geq 0 \quad \forall c \in C, m \in M \quad (3.8)$$

$$i_{s,m}^\infty = i_{s,m} - \sum_{t \in T, s_t^{dep}=s} C_{t,m} + \sum_{t \in T, s_t^{arr}=s} U_{t,m} \quad \forall s \in S, m \in M \quad (3.9)$$

$$X_{t,p} \in \{0, 1\} \quad \forall t \in T, p \in \eta(t) \quad (3.10)$$

$$C_{t,m}, U_{t,m} \in \mathbb{R}_+ \quad \forall t \in T, m \in M \quad (3.11)$$

$$D_{s,m} \in \mathbb{R}_+ \quad \forall s \in S, m \in M \quad (3.12)$$

$$Z_{t,q} \in \mathbb{R}_+ \quad \forall t \in T, q \in \rho(c) \quad (3.13)$$

Here the subsets A_t and B_t are defined as:

1. $A_t = \{t' \in T : s_{t'}^{dep} = s_t^{dep}, \tau_{t'}^+ \leq \tau_t^+\}$.
2. $B_t = \{t' \in T : s_{t'}^{dep} = s_t^{dep}, \tau_{t'}^- \leq \tau_t^+\}$.

Constraint (3.2) specifies that to each trip exactly one allowed composition is assigned. Note that the compositions of the trips before and at the start of the disruption are fixed, because these trips are already underway. For those trips the set of allowed compositions, $\eta(t)$, only consists of one composition. Constraint (3.3) state that a composition can only be assigned to the succeeding trip when it can possibly originate from the current composition and Constraint (3.4) state that the current composition can only be assigned to the trip when it can possibly originate from the previous composition.

Constraint (3.5) specifies the number of coupled train units at the end of a trip and Constraint (3.6) specifies the number of uncoupled train units at the end of a trip. Constraint (3.7) denotes that the inventory level of rolling stock type $m \in M$ at station s_t^{dep} is equal to the start inventory at the corresponding station, minus all units that are coupled from the station plus all the units that are uncoupled to the station up to the departing time of the trip. Constraint (3.8) states that the inventory must be non-negative. Constraint (3.9) specifies that the scheduled end of day balance at a station is equal to the start inventory at the corresponding station plus all units that have been uncoupled to that station, minus all units that have been coupled from that station and plus the total deviation from the scheduled end of day balance. The other constraints specify the character of the decision variables.

3.2 Extra Unit Type model

NS, and all the other operators as well, have a finite number of different rolling stock types available. The main difference between these types is the level of capacity (e.g. a VIRM6 consists of 6 carriages and an VIRM4 of 4 carriages). Multiple train units can be coupled to each other to form a train, note that different rolling stock types can be coupled to each other. Such a combination of train units is called a composition. The number of possible compositions depends on the number of different rolling stock types available and on the maximum length of the track within the stations. A track within a station has only room for a fixed number of carriages.

In the EUT model the Composition model is extended by adding a rolling stock subtype for every train unit that is scheduled to have a maintenance check somewhere during the day. Suppose NS has, for instance, 30 rolling stock units of type a and 20 units of type b available. Two units of type a require a maintenance check at station Rtd at 18.00 o'clock, one unit of type a requires a maintenance check at station Rtd at 20.00 o'clock and one unit of type b requires a maintenance check at station Asd at 19.00 o'clock. That means that the following rolling stock types are available in the model: a (27 units), b (19 units), a^* (2 units), a^{**} (1 unit) and b^* (1 unit). So, in this case three additional rolling stock types have been added to the model, while 4 units are scheduled to have maintenance during that day.

Theorem 3.1. *The number of possible compositions grows polynomially by adding more rolling stock subtypes, when the maximum size of a composition is greater than 2.*

Proof. Denote k as the maximum size of a composition, expressed in the number of units coupled to each other. Denote N as the set of different original rolling stock types available, $n = |N|$. Original rolling stock types are the types that already existed before extra subtypes for the units that require a maintenance check are added. Furthermore, denote l_j as the number of available rolling stock units of type j , $l_j \geq k$ for all original rolling stock types. So, the number of possible compositions is equal to $\sum_{i=1}^k n^i = \frac{n(n^k-1)}{n-1}$, on every place in the composition one of the original types is selected.

Adding a new rolling stock subtype p to N (e.g. because of maintenance) leads to either:

1. $l_p \geq k$, that means that the total number of possible compositions is equal to $\sum_{i=1}^k (n+1)^i = \frac{(n+1)((n+1)^k-1)}{n}$. This is polynomial in the size of n .
2. $l_p < k$. Denote $M = \{N \cup p\}$ as the set of original rolling stock types plus the new subtype p . Denote x as the increase in number of compositions when introducing the new rolling stock subtype. First note that when $l_j = 1$ we have the minimum increase in x , so if this is polynomial the increase is also polynomial for $l_j > 1$. Furthermore, note that x depends on k . For $k \leq 2$, p can either form a composition on its own (1 extra composition) or with a unit from N in the front or in the back (2 extra compositions per unit in N). So, the total increase in number of compositions is: $x = 2n + 1$ and so for $k \leq 2$ this increase is linear.

For $k \geq 3$ there are two options. Either, the new subtype p can be added to a composition existing of only identical types (e.g. aa), forming a composition of only two rolling stock types (e.g. aap). In this case, the new subtype p can replace any original type in a composition of size k . So, that leads to an increase of kn possible compositions for compositions of size k .

That means that the total increase is equal to $\sum_{i=2}^k in = \frac{k(k+1)n}{2}$ for compositions existing of only identical original rolling stock types. Secondly, there are also compositions consisting of only different types in N (e.g. abc). Note that the total number of compositions consisting of only different types in N is equal to $\sum \frac{n!}{(n-i)!}$. Adding subtype p to such a composition

thus leads to an increase of compositions equal to $\sum_{i=1}^k \frac{n!}{(n-(i-1))!} \cdot i$. For $k = 3$ we have that the increase in number of compositions of size 3 is equal to $\frac{n!}{(n-2)!} \cdot 3 > n^2$, so this is already polynomial in n . That means that for $k > 3$ the increase will also be polynomial.

□

Usually no more than 5% of the rolling stock units are scheduled for maintenance during the day and it is not preferable to schedule multiple units that require maintenance in a composition. So, a safe assumption is that every composition contains at most one maintenance unit. As a result, the increase in number of compositions only depends on the number of original rolling stock types: $\sum_{i=1}^k (n^{i-1} \cdot i) = \frac{k \cdot n^{k+1} - (k+1) \cdot n^k + 1}{(n-1)^2}$. Here, n is now fixed on the total number of original rolling stock types.

A large advantage of this approach is that by adding a rolling stock type for every maintenance unit type, maintenance constraints can easily and effectively be put on the units that require a maintenance check. To that end, denote h_m as the time maintenance unit type $m \in M$ has its maintenance appointment, g_m as the duration of the appointment, f_m as the location, ft_m as the time after which the unit may no longer be used for passenger transportation without having maintenance and n_m as the amount of units that require the maintenance check. Introduce the variable App_m as the amount of units of type $m \in M$ that have missed their maintenance appointment. Then, constraint (3.14) denotes that a maintenance unit needs to be in inventory at the time of its appointment, for the duration of its appointment and otherwise the appointment needs to be cancelled. Furthermore, constraint (3.15) states that maintenance units have to be in inventory if they miss their maintenance check before their deadline time ft_m .

$$i_{s,m} - \sum_{t' \in A_t} C_{t',m} + \sum_{t' \in B_t} U_{t',m} + App_m \geq n_m \quad \forall t \in T, m \in M, \forall s \in S$$

$$s = s_t^{dep} = f_m,$$

$$\tau_t^+ \geq h_m, \tau_t^+ \leq h_m + g_m \quad (3.14)$$

$$I_{t,m} \geq n_m - App_m \quad \forall m \in M, t \in T : \tau_t^+ \geq ft_m \quad (3.15)$$

The objective function is extended with the variable App_m , by giving a penalty on each unit that misses its maintenance appointment. Note that in real life units that are not in time for their maintenance appointment

4 The shadow problem for maintenance

The second approach to include maintenance in the rolling stock rescheduling problem is keeping trace of a shadow administration (SA). For every available train unit a 'shadow' unit is created. This shadow unit is not denoted by a rolling stock type (e.g. a or b), but by a 'maintenance' number, where a 0 stands for no maintenance needed and for every unit that requires maintenance on a different time or station, a different number ($1, \dots, x$) is appointed to the shadow unit.

For instance, consider the same situation as in the previous section, there are 30 train units of type a and 20 train units of type b . Two units of type a have a maintenance appointment at Rtd at 18:00, one unit of type a has an appointment at Rtd at 20:00 and one unit of type b has an appointment at Asd at 19:00. In the normal part of the problem there still are 30 units of type a and 20 units of type b , however in the shadow administration there are 46 units of type 0, two units of type 1, one unit of type 2 and one unit of type 3.

The number of possible extra compositions in the shadow administration depends on the number of units that require maintenance and on the maximum size of a composition k . Assuming that every composition contains maximally one unit that requires maintenance, one can easily verify that the total number of required compositions equals: $p \cdot \sum_{i=1}^k (i+1) = p(k + \frac{k^2}{2})$, where p is the number of additional subtypes.

All the constraints in the model can be decomposed into three different parts:

1. Normal part.

2. Shadow administration part.
3. Linking part.

We will explain every part separately in the following subsections.

4.1 Normal part

The normal part is exactly the same as the extra unit model. So, the normal part consists of constraints (3.2)-(3.13). Furthermore, the same objective function is used, see (3.1).

4.2 Shadow administration part

The SA part contains identical constraints as the normal part, only now to keep track of the shadow administration. To that end, denote M' as the set of maintenance types. Let a'_m be the available number of units of type $m \in M'$. P' is the set of possible SA compositions and $v'_{m,p}$ the number of train units $m \in M'$ in composition $p \in P'$. For each trip $t \in T$, $\eta'(t)$ denotes the set of allowed SA compositions.

Furthermore, let Q' be the set of allowed SA composition changes. Next, denote $\rho'(t)$ as the allowed composition changes at the end of trip $t \in T$ in the SA part. For $q \in \rho'(t)$ the incoming composition is denoted by p'_q and the outgoing composition by o'_q . Furthermore, for a given composition change $q \in Q'$, $\alpha'_{q,m}$ denotes the number of uncoupled units of type $m \in M'$ and $\beta'_{q,m}$ denotes the number of coupled units of type $m \in M'$.

$i'_{s,m}$ denotes the number of units of type $m \in M'$ in the inventory at station $s \in S$ at the start of the planning period.

The model further requires the following decision variables:

- $X'_{t,p} \in \{0, 1\}$ denotes whether composition $p \in \eta'(t)$ is used on trip $t \in T$.
- $Z'_{t,q} \in \{0, 1\}$ denotes whether composition change $q \in \rho'(t)$ is used at the end of trip $t \in T$.
- $I'_{t,m} \in \mathbb{Z}_+$ denotes the number of SA units of type $m \in M'$ in the inventory at station s_t^{dep} at time τ_t^+ .
- $C'_{t,m}$ and $U'_{t,m} \in \mathbb{Z}$ denote the number of SA units $m \in M'$ that are coupled and uncoupled at the end of trip $t \in T$.

The following constraints are required for the SA part:

$$\sum_{p \in \eta_{01}(t)} X'_{t,p} = 1 \quad \forall t \in T \quad (4.1)$$

$$X'_{\sigma(t),p} = \sum_{q \in \rho'(t): o'_q = p} Z'_{t,q} \quad \forall t \in T, p' \in \eta(t) \quad (4.2)$$

$$X'_{t,p} = \sum_{q \in \rho'(t): p'_q = p} Z'_{t,q} \quad \forall t \in T, p' \in \eta(t) \quad (4.3)$$

$$C'_{t,m} = \sum_{q \in \rho'(t)} \beta'_{q,m} Z'_{t,q} \quad \forall t \in T, m \in M' \quad (4.4)$$

$$U'_{t,m} = \sum_{q \in \rho'(t)} \alpha'_{q,m} Z'_{t,q} \quad \forall t \in T, m \in M' \quad (4.5)$$

$$I'_{t,m} = i'_{s_t^{dep},m} - \sum_{t' \in A_t} C'_{t',m} + \sum_{t' \in B_t} U'_{t',m} \quad \forall t \in T, m \in M' \quad (4.6)$$

$$I'_{t,m} \geq 0 \quad \forall c \in C, m \in M' \quad (4.7)$$

$$X'_{t,p} \in \{0, 1\} \quad \forall t \in T, m \in \eta'(t) \quad (4.8)$$

$$C'_{t,m}, U'_{t,m}, I'_{t,m} \in \mathbb{R}_+ \quad \forall t \in T, m \in M' \quad (4.9)$$

$$Z'_{t,q} \in \mathbb{R}_+ \quad \forall t \in T, q \in \rho'(c) \quad (4.10)$$

All SA constraints operate in the same way as the constraints in the normal part.

4.3 Linking part

Maintenance constraints will be set on the SA units that require maintenance (the types $1, \dots, x$). As a result, paths will be created for the maintenance types to their maintenance location. Such a path has to be present for its corresponding original type (e.g. a or b in the normal part as well). That means that a link between the two parts is necessary. Such a link is two-fold. First of all, the solution of both parts need to be the equal in terms of:

- the length of compositions assigned to a trip.
- the number of coupled/ uncoupled rolling stock units.
- the numbers of normal and SA train units in inventory.
- the length of the incoming and outgoing composition in a chosen composition change.

That means that constraints (3.2)-(3.7) of the normal part are linked to constraints (4.1)-(4.6) of the SA part. Secondly, a link must be constructed to connect a SA unit of, for instance, type 1, that requires maintenance to a normal unit of, for instance, type a .

Theorem 4.1. *Only constraint (4.11) is needed to link the size, coupling/uncoupling, inventory and composition changes of the normal and SA part.*

$$\sum_{\substack{p \in \eta(t) \\ n_p = r}} X_{t,p} = \sum_{\substack{p \in \eta'(t) \\ n_p = r}} X'_{t,p} \quad \forall t \in T, r \in R \quad (4.11)$$

Proof. We claim that by using (4.11) the variables $U_{t,m}$ & $U'_{t,m}$, $C_{t,m}$ & $C'_{t,m}$, $I_{t,m}$ & $I'_{t,m}$ and $Z_{t,q}$ & $Z'_{t,q}$ are linked in the normal and SA part. We will prove this in steps.



Figure 2: Connection between two trips

- First, $U_{t,m}$ and $U'_{t,m}$. We will show that $\sum_{m \in M} U_{t,m} = \sum_{m \in M'} U'_{t,m} \forall t \in T$ by contradiction. Assume that $\sum_{m \in M} U_{t,m} > \sum_{m \in M'} U'_{t,m}$ for at least one trip. At the end of trip $t \in T$ more units are coupled in the normal part than in the SA part.

See Figure 2 for a visualization of the situation. By definition we have that $\sum_{\substack{p \in \eta(t) \\ n_p = r}} X_{t,p} = \sum_{\substack{p \in \eta'(t) \\ n_p = r}} X'_{t,p}$, that means that the length of the compositions assigned to trip t_1 and t_2 are

equal in the normal part and in the SA part. It is assumed that more units are coupled at the end of trip t_1 in the normal part than in the SA part. This is only possible if also more units are uncoupled in the normal part. However, by definition it is not allowed to both couple and uncouple units at a connection. This leads to a contradiction and so $\sum_{m \in M} U_{t,m} \leq \sum_{m \in M'} U'_{t,m}$.

The same proof holds in the other direction, namely that $\sum_{m \in M'} U'_{t,m} \leq \sum_{m \in M} U_{t,m}$. We can conclude that

$$\sum_{m \in M'} U'_{t,m} = \sum_{m \in M} U_{t,m}, \text{ if } \sum_{\substack{p \in \eta(t) \\ n_p = r}} X_{t,p} = \sum_{\substack{p \in \eta'(t) \\ n_p = r}} X'_{t,p} \forall t \in T$$

- $C_{t,m}$ and $C'_{t,m}$, use the same proof as for $U_{t,m}$ and $U'_{t,m}$ to find that:

$$\sum_{m \in M'} C'_{t,m} = \sum_{m \in M} C_{t,m} \quad \forall t \in T$$

- $I_{t,m}$ and $I'_{t,m}$. Assume that $\sum_{m \in M} I_{t,m} > \sum_{m \in M'} I'_{t,m}$ at the start of at least one trip $t \in T$. Furthermore, the inventory at the start of the day is given and it holds that:

$$\sum_{m \in M} I_{s,m} = \sum_{m \in M'} I'_{s,m}$$

So, a difference between $I_{t,m}$ and $I'_{t,m}$ arises during the operations. Note that $I_{t,m} = I_{s,m} - \sum_{t' \in T: t' \in A_t} C_{t',m} + \sum_{t' \in T: t' \in B_t} U_{t',m}$. Meaning that a difference between $I_{t,m}$ and $I'_{t,m}$ can only be caused by either $C_{t,m}$ or $U_{t,m}$, but we just showed that $\sum_{m \in M} C_{t,m} = \sum_{m \in M'} C'_{t,m}$ and

$\sum_{m \in M} U_{t,m} = \sum_{m \in M'} U'_{t,m}$. So, it holds that

$$\sum_{m \in M} I_{t,m} = \sum_{m \in M'} I'_{t,m}$$

- $Z_{t,q}$ and $Z'_{t,q}$ are the same in size when the length of the incoming compositions p_q and p'_q are equal and the length of the outgoing compositions o_q and o'_q are equal as well. By definition we have that the length of appointed compositions to a trip are equal, so it holds by definition that $Z_{t,q}$ and $Z'_{t,q}$ are equal.

□

Furthermore, there have to be constraints to link the flow of a unit that requires maintenance to the flow of its corresponding original unit. In other words, when a SA unit of type 1 resides in a SA composition on place i then a unit a must reside in the normal composition on place i as well. Also, when a unit of type 1 is coupled (uncoupled), a normal unit a must be coupled (uncoupled) as well. Finally, as long as a unit of type 1 is in inventory, a unit of type a must be in inventory as well. To this end, denote n as the maximum size of a possible composition ($n = \max(\sum_{m \in M} v_{m,p})$) and $w_{i,p}$ as the unit assigned to place $i \in \{1, \dots, n\}$ in composition $p \in P$. Constraints (4.12) and (4.13) are added to the model in order to link a SA unit of type 1 to a normal unit of type a . Here, constraint (4.12) states that when a unit of type 1 resides in a SA composition appointed to trip $t \in T$ on place i , then a unit of type a must reside in the normal composition appointed to trip t on the same place. Constraint (4.13) specifies that when a unit of type 1 is in inventory at station s , then at least one corresponding unit of type a must be in inventory at s .

$$\sum_{\substack{p \in P': \\ w_{i,p}=1}} X'_{t,p} \leq \sum_{\substack{p \in P: \\ w_{i,p}=a}} X_{t,p} \quad \forall t \in T, i \in \{1, \dots, n\} \quad (4.12)$$

$$I'_{t,m'} \leq I_{t,m} \quad \forall t \in T, m' \in M' = 1, m \in M = "a" \quad (4.13)$$

In the same way other units that require maintenance can be linked to their original units.

Furthermore, constraints are required to force a maintenance unit to be on time for its appointment. To this end, introduce the variable $App_{m'}$ denoting the amount of rolling stock units of type $m' \in M'$ missing their maintenance appointment and the parameters $n_{m'}$ as the total number of units available of type $m' \in M'$, $f_{m'}$ as the station at which m' has its appointment, $g_{m'}$ as the time of the appointment, $h_{m'}$ as the duration of the appointment, n'_m as the amount of units that require the maintenance check, and ft'_m as the deadline time of units of type $m' \in M'$. Then, Constraint (4.14) specifies that either a maintenance unit is present at the station where its maintenance appointment is scheduled at the right time for the right duration, or that the maintenance appointment is cancelled. The objective function is then extended with the variable $App_{m'}$, giving a penalty on each unit that misses its appointment. Furthermore, constraint (4.15) states that maintenance units have to be in inventory if they miss their maintenance check before their deadline time ft'_m .

$$I'_{s,m'} - \sum_{t' \in A_t} C'_{t',m'} + \sum_{t' \in B_t} U'_{t',m'} + App_{m'} \geq n_{m'} \quad \forall t \in T, m' \in M', s \in S:$$

$$s_t^{dep} = s = f_{m'}, \tau_t^+ \geq g_{m'},$$

$$\tau_t^+ \leq g_{m'} + h_{m'} \quad (4.14)$$

$$I'_{t,m'} \geq n'_m - App'_{m'} \quad \forall m' \in M', t \in T : \tau_t^+ \geq ft'_m \quad (4.15)$$

5 Job-Composition model

In this section the Job-Composition model to take maintenance into account in the RSRP is introduced. In a rolling stock circulation, a particular unit is being coupled to a certain departing trip and fulfills a number of successive trips until the particular unit is being uncoupled. A job is such a sequence of successive trips. Rolling stock units will be appointed to jobs in such a way that as much of the passenger service as possible is upheld. First we start with extending the notation of the previous sections

We stress two subsets of Q (the set of composition changes), namely Q^+ and Q^- denoting composition changes with only coupling (Q^+) or uncoupling (Q^-) activities.

Denote J as the set of possible jobs, let $E(j)$ be the set of trips covered by job $j \in J$. Every job $j \in J$ has a start (and final) trip, this is the first (last) trip covered by j , denoted by λ_j (γ_j). Furthermore, denote ω_j (π_j) as the side at which job $j \in J$ is coupled (uncoupled) (L or R).

During the whole day jobs are being performed by train units. When a disruption occurs, there are obviously still jobs underway. It is not allowed to change the trips that are already underway. Denote the set of trips that have started before the disruption and are still operational by $D(t)$ and $G_{t,p}$ as the composition $p \in P$ that is assigned to trip $t \in D(t)$.

Finally, denote MR as the set of units that have a maintenance appointment somewhere during the day. Here, a_r is the rolling stock type of $r \in MR$, l_r is the location of unit r at the start of the disruption (either a station or a trip), f_r is the station at which unit r has its maintenance appointment, g_r is the time at which the appointment takes place and h_r is the duration of the appointment.

Then, we need the following variables in our model:

- $X_{t,p} \in \{0, 1\}$ denotes whether composition $p \in P$ is used on trip $t \in T$.
- $N_t \in \{0, 1\}$ denotes whether trip $t \in T$ is cancelled.
- $Z_{t,q} \in \{0, 1\}$ denotes whether composition change $q \in Q$ is used at the end of trip $t \in T$.
- $W_j \in \{0, 1\}$ denotes whether job $j \in J$ is used or not.
- $Y_{j,m} \in \{0, 1\}$ denotes whether job $j \in J$ is covered by a rolling stock unit of type $m \in M$.
- $Q_{j,r} \in \{0, 1\}$ denotes whether job $j \in J$ is covered by a maintenance unit $r \in R$.
- $C_{t,m}$ and $U_{t,m} \in \mathbb{R}^+$ denotes the number of coupled and uncoupled units of type $m \in M$ at the end of trip $t \in T$.
- $I_{t,m} \in \mathbb{R}^+$ denotes the inventory of type $m \in M$ at station $s(t)$ just after trip $t \in T$ departs.
- $I_{s,m}^\infty \in \mathbb{R}^+$ denotes the inventory of type $m \in M$ at station $s \in S$ of type at the end of the planning period.
- $D_{s,m}$ denotes the deviation from the desired end of day balance on station $s \in S$ for rolling stock type $m \in M$.
- $App_r \in \{0, 1\}$ denotes whether unit $r \in AR$ is on time for its maintenance appointment.

5.1 Model

$$\min f(N, X, D, Z, App) \quad (5.1)$$

$$\sum_{j \in J: E(j) \ni t} W_j + N_t \geq 1 \quad \forall t \in T \quad (5.2)$$

$$\sum_{m \in M} Y_{j,m} = W_j \quad \forall j \in J \quad (5.3)$$

$$C_{t,m} = \sum_{j \in J: \lambda_j = t} Y_{j,m} \quad \forall t \in T, m \in M \quad (5.4)$$

$$U_{t,m} = \sum_{j \in J: \gamma_j = t} Y_{j,m} \quad \forall t \in T, m \in M \quad (5.5)$$

$$I_{t,m} = i_{s_t}^0 - \sum_{\substack{t' \in T \\ s_{t'}^{dep} = s_t^{dep} \\ \tau_{t'}^{dep} \leq \tau_t^{dep}}} C_{t,m} + \sum_{\substack{t' \in T \\ s_{t'}^{arr} = s_t^{dep} \\ \tau_{t'}^{arr} \leq \tau_t^{dep} - \rho}} U_{t,m} - \sum_{\substack{r \in R \\ a_r = m \\ \tau_t^{dep} \geq g_r \\ \tau_t^{dep} \leq h_r \\ s_t^{dep} = f_r}} 1 \quad \forall t \in T, m \in M \quad (5.6)$$

$$I_{s,m}^\infty = i_{s,m}^0 - \sum_{t \in T: s(t)=s} C_{t,m} + \sum_{t \in T: s(t)=s} U_{t,m} + D_{s,m} \quad \forall s \in S, m \in M \quad (5.7)$$

$$\sum_{\substack{j \in J \\ \lambda_j = \sigma(t) \\ \pi_j = \eta_q}} Y_{j,m} = \sum_{q \in Q^+} Z_{t,q} \beta_{q,m} \quad \forall m \in M, t \in T \quad (5.8)$$

$$\sum_{\substack{j \in J \\ \gamma_j = t \\ \pi_j = \eta_q}} Y_{j,m} = \sum_{q \in Q^-} Z_{t,q} \alpha_{q,m} \quad \forall m \in M, t \in T \quad (5.9)$$

$$W_j + W_{j'} \leq 1$$

$$\forall j, j' \in J : R_j \cap R_{j'} \neq \emptyset, \\ \omega_j = \pi_{j'} \text{ and}$$

$$\tau_j^{dep} < \tau_{j'}^{dep} < \tau_j^{arr} < \tau_{j'}^{arr} \quad (5.10)$$

$$W_j + W_{j'} \leq 1$$

$$\forall j, j' \in J : R_j \cap R_{j'} \neq \emptyset, \\ \omega_{j'} \neq \pi_{j'} \text{ and}$$

$$\tau_{j'}^{dep} < \tau_j^{dep} < \tau_j^{arr} < \tau_{j'}^{arr} \quad (5.11)$$

$$I_{t,r} = i_{s_t}^0 - \sum_{\substack{t' \in T \\ s_{t'}^{dep} = s_t^{dep} \\ \tau_{t'}^{dep} \leq \tau_t^{dep}}} \sum_{\substack{j \in J \\ \lambda_j = t}} Q_{j,r} - \sum_{\substack{t' \in T \\ s_{t'}^{arr} = s_t^{dep} \\ \tau_{t'}^{arr} \leq \tau_t^{dep} - \rho}} \sum_{\substack{j \in J \\ \gamma_j = t}} Q_{j,r}$$

$$\forall t \in T, r \in AR \quad (5.12)$$

$$\begin{aligned}
I_{t,r} &\geq 1 - App_r & \forall r \in AR, t \in T : \\
& & \tau_t^{dep} \geq g_r, \\
& & \tau_t^{dep} \leq h_r + g_r, \\
& & s_t^{dep} = f_r \tag{5.13}
\end{aligned}$$

$$Q_{j,r} \leq Y_{j,m} \quad \forall j \in J, r \in AR, m \in M : a_r = m \tag{5.14}$$

$$I_{t,r} \leq I_{t,m} \quad \forall t \in T, r \in AR, m \in M : a_r = m \tag{5.15}$$

$$X_{t,p} = \sum_{\substack{q \in \rho(t) \\ p_q = p}} Z_{t,q} \quad \forall t \in T, p \in \eta(t) \tag{5.16}$$

$$X_{\sigma(t),p} = \sum_{\substack{q \in \rho(t) \\ o_q = p}} Z_{t,q} \quad \forall t \in T, p \in \eta(t) \tag{5.17}$$

$$X_{t,p} = G_{t,p} \quad \forall t \in D(t), p \in P \tag{5.18}$$

$$X_{t,p} \in \{0, 1\} \quad \forall t \in T, p \in P \tag{5.19}$$

$$W_d \in \{0, 1\} \quad \forall j \in J \tag{5.20}$$

$$Z_{t,q} \in \mathbb{R}^+ \quad \forall t \in T, q \in Q \tag{5.21}$$

$$Y_{j,m} \in \{0, 1\} \quad \forall j \in J \tag{5.22}$$

$$I_{t,m}, C_{t,m}, U_{t,m} \in \mathbb{R}^+ \quad \forall t \in T, m \in M \tag{5.23}$$

$$I_{s,m}^\infty \in \mathbb{R}^+ \quad \forall s \in S, m \in M \tag{5.24}$$

$$I_{t,r} \in \mathbb{R}^+ \quad \forall t \in T, r \in AR \tag{5.25}$$

$$Q_{j,r} \in \mathbb{R}^+ \quad \forall j \in J, r \in AR \tag{5.26}$$

5.2 Explanation of the constraints

Constraint (5.2) states that at least one job covers trip $t \in T$ or else the trip is cancelled. Constraint (5.3) states that every chosen job is performed by a certain rolling stock type $m \in M$. Constraint (5.4) states that the number of coupled units of type $m \in M$ at the start of trip $t \in T$ is equal to all the units m that start their job at trip t . Constraint (5.5) states that the number of uncoupled units $m \in M$ at the end of trip $t \in T$ is equal to the number of units m that finish their job at the end of trip t .

Constraint (5.6) specifies that the inventory at station s_t^{dep} at the start of trip $t \in T$ is equal to the inventory at s_t^{dep} at the start of the disruption minus all the units that have been coupled from the station up to the departing time of t , plus all the units that have been uncoupled to $s^{dep}(t)$ before the departing time minus the slack time needed to uncouple and transport the unit to the shunting area ($\tau_t^{dep} - \rho$), and finally minus all the units that have been uncoupled to s_t^{dep} but who undergo a maintenance check at the time of the departure of trip t . The end inventory at station $s \in S$ is equal to the inventory at the start of the disruption on the corresponding station minus all the units that have been coupled from the station and plus all the units that have been uncoupled to the station, as can be seen in Constraint (5.7).

Constraint (5.8) states that a composition change $q \in Q^+$ can only take place at the end of trip $t \in T$ if $\beta_{q,m}$ units of type $m \in M$ start their job at the successive trip σ_t . Constraint (5.9) states that a composition change $q \in Q^-$ can only take place at the end of trip $t \in T$ if $\alpha_{q,m}$ units of type $m \in M$ end their job at trip t . Furthermore, constraint (5.10) states that uncoupling of job $j \in J$ at side π_j^a is not allowed when there is a job $j' \in J$ that started later, but that covers at least one identical trip as job j and that is coupled at side π_j^a . See Figure 5.2 as an example: a unit in job j is driving first on its own, then a unit of job j' is coupled to it at the right side. From that moment on it is not allowed to uncouple the unit j at the right side until job j' is uncoupled.



Figure 3: Example of constraint

Constraint (5.11) states that uncoupling of job $j' \in J$ at side $\pi_{j'}^d$ is not allowed when there is a different job $j \in J$ that started earlier, but that covers at least one identical trip as job j' and that is not coupled at side π_j^a . See Figure 5.2 again as an example: a unit in job j is driving first on its own, then a unit of job j' is coupled to it at the right side. From that moment on it is not allowed to uncouple the unit j' at the left side, unless job j is uncoupled.

Constraint (5.12) keeps track of the inventory of maintenance units $r \in R$ at the beginning of trip $t \in T$. This is equal to the start inventory of $r \in R$ at the corresponding station minus all maintenance units r that have been coupled from the station and plus all the maintenance units r that have been uncoupled to the station up to trip t . Constraint (5.14) is comparable to the linking constraint in the shadow account model. If a maintenance unit with type a_r is used on job $j \in J$ then the corresponding unit of type $m \in M$ must also be appointed to job $j \in J$. The same holds for the inventory, if a unit of type a_r is in inventory, then the corresponding unit of type $m \in M$ must also be in inventory, as is shown in Constraint (5.15). Constraint (5.13) states that every maintenance unit must be in inventory at the start of their appointment, at the right location and for the right duration or else the appointment is cancelled.

Constraint (5.16) states that the composition appointed to trip $t \in T$ is equal to the incoming composition of the composition change of trip t and constraint (5.17) states that composition appointed to σ_t is equal to the outgoing composition of the composition change of trip t . Constraint (5.18) specifies that to every trip $t \in D(t)$ that is underway at the start of the disruption the original composition must be appointed.

Finally, Constraints (5.19)-(5.26) specifies the domains of the variables.

6 Results

In this section we discuss the results of applying the Extra Unit Type model and the Shadow Account model on a real life case from NS. So far, no results are available for the Job-Composition model. The data is from the 3000 and 2200 line where trains are travelling from Den Helder (Hdr) to Nijmegen (Nm) (3000 line) and from Dordrecht (Ddr) to Amsterdam (Asd) (2200 line), see Figure 4.

Different disruptions are simulated between 07:00- 09:00, either between Rotterdam and the Hague, between Utrecht and Amsterdam or between Amsterdam and Haarlem. Only two rolling stock types are considered, namely type a and b . Each case is ran twice, in the first case a unit of type a consists of 4 carriages and a unit of type b of 6 carriages and in the second case a unit of type a consists of 3 carriages and a unit of type b of 4 carriages. In the first case the maximum number of carriages on a line is equal to 12 and in the second case equal to 15. So, in the first case there are 8 different compositions and in the second case 31. The tests are conducted with Cplex 12.5 on an Intel(R) i5 core with 2.50 GHz and 8 GB RAM.

In total 50 different simulations were ran. The results are summarized in Table 6. The first column denotes which model is used, either the Original model, the Extra Unit Type model or the Shadow Account model. In the second column the number of carriages is denoted, either 4 and 6, or 3 and 4. In the third column the average number of composition changes is shown, there is an average because we take a different number of maintenance units into account each simulation. In the final column the total computation time is denoted. As can be seen the extra unit type model is better when there are little possible composition changes and the Shadow Account model is better with many possible composition changes. Note that both models give the same solution. In comparison with the original model, both models are significant slower, however the units will



Figure 4: Example lines in the Netherlands

Model	Carriages	# Composition changes	CPU
Original	4,6	94	5s
Extra Unit Type	4,6	231	35s
Shadow	4,6	279	60s
Original	3,4	416	7,30s
Extra Unit Type	3,4	1415	3100s
Shadow	3,4	815	350s

Table 1: Results

miss their maintenance appointment when using the original model.

7 Conclusions and further research

In this paper we extended the Rolling Stock Rescheduling problem by taking maintenance appointments into account. The first extension is the Extra Unit Type model, where an extra type is added for every unit that requires maintenance. The second extension is the Shadow Account model, where a shadow account is being kept for the units that require maintenance. The final model is the Job-Composition model where jobs are appointed to rolling stock units.

The results show that the Extra Unit Type model performs slightly better when there are little possible composition changes, however when there are more possible composition changes the Shadow Account model outperforms the Extra Unit Type model.

In further research, computational results of the Job-Composition model will be presented, as well as an extension of the results of the other two models.

References

- [1] P. Fioole, L. Kroon, G. Maróti, and A. Schrijver. A rolling stock circulation model for combining and splitting of passenger trains. *European Journal Of Operational Research*, 174: 1281–1297, 2006.

- [2] L.G. Kroon, G. Maróti, and L. Nielsen. Rescheduling of railway rolling stock with dynamic passenger flows. *Technical Report, No. ERS-2010-045-LIS, Erasmus Research Institute of Management, Rotterdam*, 2010.
- [3] G. Maróti and L.G. Kroon. Maintenance routing for train units: the transition model. *Transportation Science*, 39(4):518–525, 2005.
- [4] G. Maróti and L.G. Kroon. Maintenance routing for train units: the interchange model. *Computers & Operations Research*, 34(4):1121–1140, 2007.
- [5] L. Nielsen, L.G. Kroon, and G. Maróti. A rolling horizon approach for disruption management of railway rolling stock. *European Journal of Operational Research*, 220:496–509, 2012.
- [6] L.K. Nielsen. Rolling stock rescheduling in passenger railways. *Ph.D thesis, Erasmus University, Rotterdam, The Netherlands*, 2011.